STAT 327

Solutions to Assignment #5

8.46 $n \geq z^2_{\alpha/2}(1/4)/E^2$ $\quad z_{\alpha/2} = z_{.025} = 1.960$, $E = 0.07$

So $n \geq \dfrac{(1.96)^2}{4(.07)^2} \doteq \underline{\underline{196}}$

8.47 $n \geq \dfrac{z^2_{.025}\,\hat{p}(1-\hat{p})}{(.05)^2} = \dfrac{(1.96)^2(.44)(.56)}{(.05)^2} = 378.6$

So $n = \underline{\underline{379}}$.

8.50 $n \geq \dfrac{z^2_{.025}(.48)(.52)}{(.025)^2} \approx \underline{\underline{1534}}$

8.51 (a) $n \geq \dfrac{\sigma^2 z^2}{m^2}$, $\sigma = 200$, $m = 25$
$z = z_{.025} = 1.96$

$n \geq \dfrac{(200)^2(1.96)^2}{(25)^2} = 245.86$, so $\underline{\underline{n = 246}}$

(b) margin of error $= z_{\alpha/2}\dfrac{s}{\sqrt{n}} = 1.96\dfrac{300}{\sqrt{246}} = \underline{\underline{37.48}}$

8.97 (a) Assumption: the standard deviation will be similar now

$n \geq \dfrac{(1.96)^2(1000)^2}{(100)^2} = 384.16$, so $n = \underline{\underline{385}}$

(b) more than \$100 because the standard error will be larger than predicted. **(c)** With a larger margin of error, the 95% confidence interval is wider; thus, the probability that the sample mean is within \$100 of the population mean is less than 0.95.

9.9 $H_0: p = 1/6$

$H_a: p > 1/6$

9.10 Let p = true probability of a correct prediction

$H_0: p = 1/4$

$H_a: p > 1/4$

9.11 (a) $P[Z > 1.04] = 1 - .8508 = \underline{\underline{0.1492}} \approx \underline{\underline{0.15}}$

(b) P-value $= 2P[Z > 1.04] = 2(.15) = \underline{\underline{.30}}$

(c) P-value $= P[Z < 1.04] = \underline{\underline{0.85}}$

(d) No. All of them indicate that the null hypothesis is plausible. (a) gives the least weakest evidence against H_0.

9.12 (a)(i) P-value $= P[Z > 2.50] = \underline{\underline{.0062}} \approx \underline{\underline{.006}}$

(ii) $2P[Z > 2.5] = 2(.006) = \underline{\underline{.012}}$

(iii) $P[Z < 2.5] = 1 - .006 = \underline{\underline{.994}}$

(b) Yes — those in (i) and (ii) are very small.

9.14 (a) H_0: $p = 1/5$ **(b)** H_a: $p \neq 1/5$ **(c)** H_a: $p > 1/5$
(d) P-value ≈ 0. The probability of obtaining a sample proportion of 81 or more successes in 83 trials is essentially 0.

9.18 (a) $\hat{p} = 22/30 = 0.733$; $se = \sqrt{p_0(1 - p_0)/n} = \sqrt{0.50(1 - 0.50)/30} = 0.091$; $z = (0.733 - 0.5)/0.091 = 2.56$ **(b)** Area beyond $z = 2.56$ is 0.005, doubling to include both tails gives a P-value of 0.01. If the null hypothesis were true, the probability would be 0.01 of getting a test statistic at least as extreme as the value observed. **(c)** The data must be categorical and obtained using randomization, and the sample size must be large enough that the sampling distribution is approximately normal. In this case, the data were obtained from a convenience sample which might not be representative of the population.

9.22 (a) Variable = whether or not one voted for Webb, parameter = p = population proportion of Virginia votes for Webb **(b)** H_0: $p = 0.50$; H_a: $p \neq 0.50$; voting status for Webb is categorical, the sample is random, the sample is large enough to assume that the sampling distribution is approximately normal: $np = (2011)(0.5) \geq 15$ and $n(1 - p) = (2011)(0.5) \geq 15$. **(c)** P-value = 0.92. Assuming $p = 0.50$, the probability of obtaining a sample proportion where 50.1% or more of the voters voted for Webb or the other extreme, less than 49.9% (since two sided alternative) is about 92%. **(d)** Since the P-value > 0.05, there is insufficient evidence to predict who won the election.

9.28 Using Minitab, we obtain:

(a) $2P[t_{19} > 2.40] \approx .026$

(b) $P[t_{19} > 2.40] \approx .013$

(c) $P[t_{19} < 2.40] = 1 - .013 = .987$.

Without using Minitab, here is the best we can do on part (a) using the t-table in the text:

From the table $P[t_{19, .025} = 2.093]$, so $P[t_{19} > 2.093] = .02$

and $P[t_{19, .010} = 2.539]$, so $P[t_{19} > 2.539] = .0$

Thus $.010 < P[t_{19} > 2.40] < .025$, so $\boxed{0.20 < \text{P-value} < 0.50}$.

9.29 (a) larger — the t-value of 1.20 is less extreme.
(b) because larger sample sizes decrease the standard error (by making its denominator larger).

9.31 **(a)** The variable is the number of hours worked in the previous week by male workers; the parameter is the population mean work week (in hours) for men. **(b)** H_0: $\mu = 40$; H_a: $\mu > 40$. **(c)** P-value ≈ 0. The P-value is the probability of obtaining a sample with a mean of 45.3 or more hours if the null hypothesis were true. **(d)** Since the P-value is less than the significance level of 0.01, there is sufficient evidence to reject the null hypothesis and to conclude that the population mean work week for men exceeds 40 hours. **9.32** **(a)** Variable = number of hours worked in the previous week by workers aged 18-25; parameter = the population mean work week (in hours) for workers aged 18-25. **(b)** H_0: $\mu = 40$; H_a: $\mu > 40$. **(c)** $t = -2.32$, the sample mean is 2.32 standard errors below the hypothesized mean. **(d)** P-value = 0.02. The P-value is the probability of obtaining a sample mean at least as far from 40 (in either direction) as the observed sample mean of 37.8, if the null hypothesis were true. For $\alpha = 0.05$, we reject the null hypothesis and conclude that the population mean work week for 18-25-year-old workers differs from 40 hours. **9.33** **(c)** The P-value of 0.046 is smaller than 0.05, so we have enough evidence to reject the null hypothesis. There is relatively strong evidence that the wastewater limit is being exceeded. **(d)** If it would be unusual to get a sample mean of 2000 if the population mean were 1000, it would be even more unusual to get this sample mean if the population mean were less than 1000.

9.39 **(b)** Most of the data fall between 4 and 14. The sample size is small so it we cannot tell too much from the plot, but there is no evidence of severe non-normality. **(c)** **(1)** data are quantitative, produced randomly; population distribution approximately normal. **(2)** H_0: $\mu = 0$; H_a: $\mu \neq 0$ **(3)** $t = 4.19$ **(4)** P-value = 0.001 **(5)** Strong evidence against the null hypothesis that family therapy has no effect.

9.43 (a) $\alpha = .05$, since the "significance level" is defined as the prob of Type I error.
(b) Since P-value $= .01 < \alpha (= .05)$, our decision is to Reject H_0. So the only possible we could have made would be to reject H_0 when H_0 is true — that is, a Type I error.

9.47 **(a)** If H_0 is rejected, we conclude that the defendant is guilty. **(b)** A Type I error would result in finding the defendant guilty when he/she is actually innocent. **(c)** If we fail to reject H_0, the defendant is found not guilty. **(d)** A Type II error would result in failing to convict a defendant who is actually guilty.

9.50 **(a)** Detect prostate cancer when there is none; results in treatment or further testing for patient who does not need it. **(b)** Fail to detect prostate cancer when it is present; results in a patient not receiving needed care. **(c)** Type II error. **(d)** The 2/3 refers to the probability that someone does not have prostate cancer given that he received a positive result. The probability of a Type I error, refers to the probability that someone will receive a positive result, given that he does not have prostate cancer.

9.79 (a) (i) $\Sigma x_i = 3+7+3+3+0+8+1+12+5+8$
$= 50$,

so $\bar{x} = \Sigma x_i / n = 50/10 = \underline{5}$

(ii) $\Sigma x_i^2 = 9+49+9+9+0+64+1+144+25+64$
$= 374$,

$$\text{so } s^2 = \frac{\Sigma x_i^2 - (\Sigma x_i)^2/n}{n-1} = \frac{374 - (50)^2/10}{9}$$

$$= \frac{124}{9} = 13.77778,$$

$$\text{so } s = \sqrt{13.7778} = \underline{3.7118}$$

(iii) $se = s/\sqrt{n} = \frac{3.7118}{\sqrt{10}} = \underline{1.174}$

(iv) $df = n-1 = 10-1 = \underline{9}$.

9.79 **(b)** sufficient evidence to reject null and conclude that the population mean is not 0 **(c)** 0.001; we have strong evidence to conclude that the population mean is positive. **(d)** 0.999; insufficient evidence to conclude that the population mean is negative.

10.2 **(a)** dependent because both samples consisted of the same 1010 adults **(b)** independent because the samples consist of different adults

10.3 **(a)** 0.07; the percentage appeared to have icreased between 1993 and 2001. **(b)** 0.043; standard error is standard deviation of sampling distribution of difference between the sample proportions. **(c)** (−0.01, 0.15); it contains zero, so it is plausible that there is no change between 1993 and 2005 in the population proportion of UW students who reported binge drinking at least 3 times in the past 2 weeks. **(d)** data are categorical (stated "getting drunk" vs. did not), samples are independent and obtained randomly, sufficiently large sample sizes.

Detail of calculation in 10.3:

(a) $0.382 - 0.312 = 0.070$

(b) $se = \sqrt{\frac{(.382)(.618)}{495} + \frac{.312(.688)}{159}} = 0.043$

(c) $0.07 \pm 1.96\,(0.043)$ yield $-0.01 < p_2 - p_1 < 0.15$.

10.4 **(a)** 0.042; there was a decline of about 4% in current smokers between 1991 and 2003 **(b)** We can be 99% confident that the population proportion for current smokers in 1991 is between 0.03 and 0.05 larger than the population proportion for current smokers in 2003. **(c)** categorical data, independent samples that are obtained randomly, each sample is large enough to have at least ten "successes" and ten "failures"

10.16 **(a)** 32.6 − 18.1 = 14.5 **(b)** *se* = 0.297; large sample sizes **(c)** We can be 95% confident that the difference between the population mean scores of women and men falls between 13.9 and 15.1. Because zero is not in the interval, we can conclude that the population mean for women is higher than the population mean for men. **(d)** The data are quantitative, both samples are independent and random, and there is approximately a normal population distribution for each group.

The calculation in 10.16 (b) is: $se = \sqrt{\frac{(18.2)^2}{6764} + \frac{(12.9)^2}{4252}}$.

10.20 (−\$14.05, \$17.97). We can be 95% confident that the difference between the population mean prices resulting from buy-it-now auctions and bidding only auctions falls between −\$14.05 and \$17.97. Because zero is in the interval, there may be no difference.

10.29 **(a)** Let group 1 represent the students who planned to go to graduate school and group 2 represent those who did not. Then, $\bar{x}_1 = 11.67$, $s_1 = 8.34$, $\bar{x}_2 = 9.10$ and $s_2 = 3.70$. The sample mean study time per week was higher for the students who planned to go to graduate school, but the times were also much more variable for this group. **(b)** $se = 2.16$. If further random samples of these sizes were obtained from these populations, the differences between the sample means would vary. The standard deviation of these values would equal about 2.2. **(c)** A 95% confidence interval is $(-1.9, 7.0)$. We are 95% confident that the difference in the mean study time per week between the two groups is between −1.9 and 7.0 hours. Since 0 is contained within this interval, we cannot conclude that the population mean study times differ for the two groups.

Note: As described in the text, the formulas used for the small-sample confidence intervals in problems 10.20 and 10.29 were:

$$\bar{x}_1 - \bar{x}_2 \pm t_{df, \alpha/2} \cdot se, \text{ where}$$

$$se = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$ and "df" is selected somehow by software. I do not expect you to be able to do this kind of problem by hand.

10.34 **(a)** $se = 1.025$ **(b)** $(-4.9, -0.7)$. We can be 95% confident that the population mean difference is between −4.9 and −0.7. Because 0 does not fall in this interval, we can conclude that, on average, the sexually abused students had a lower population mean family cohesion than the non-abused students did.

10.35 **(a)** P-value $= 0.011$ **(b)** data are quantitative; random samples from two groups; populations have approximately normal distributions; population standard deviations are equal. Normality assumption is likely violated, but we're using a 2-sided test, so inferences are robust to that assumption.

(Calculation details for 10.34 & 10.35 appear on next page).

For problems 10.34 and 10.35, assume normal distributions with unknown mean and variances for the two groups, but with $\sigma_1 = \sigma_2 = \sigma$ (unknown).

10.34 (a) $\bar{X}_1 = 2.0$, $n_1 = 13$, $s_1 = 2.1$
$\bar{X}_2 = 4.8$, $n_2 = 17$, $s_2 = 3.2$

$$se_{\bar{x}_1 - \bar{x}_2} = s\sqrt{\frac{1}{13} + \frac{1}{17}},$$

where $s = \sqrt{\dfrac{12(2.1)^2 + 16(3.2)^2}{28}}$

(b) 95% CI for $\mu_1 - \mu_2$ is $2.0 - 4.8 \pm t_{28,.025} \cdot se$

OR $-2.8 \pm 2.048(1.025)$

OR $-4.9 < \mu_1 - \mu_2 < -0.7$.

10.35 (a) test statistic $t = \dfrac{\bar{x}_1 - \bar{x}_2}{se_{\bar{x}_1 - \bar{x}_2}} = \dfrac{-2.8}{1.025} = -2.73$

P-value $= 2P[t_{28} < -2.73] = 2P[t_{28} > 2.73]$.

We need to use Minitab to obtain P-value $= 0.011$.

Using the t-table, we see that $P[t_{28} > 2.763] = .005$, so that the P-value must be a little bit more than $2(.005) = .010$.