STAT 327

Solutions to Assignment #6

(1) This is clearly a "paired comparisons" problem, where the variable of interest is the difference between the peak wind gusts in January and in April at each station. The data $d_i = x_i - y_i$, $i = 1, \dots, 5$ is:

$d_1 = 139 - 104 = 35$, $d_2 = 122 - 113 = 9$, $d_3 = 26$,

$d_4 = -24$ and $d_5 = 17$.

Test $H_0: \mu_d = 0$
vs $H_a: \mu_d > 0$ at level $\alpha = 0.01$.

Under the assumption $d_1, d_2, d_3, d_4, d_4 \overset{indep}{\sim} N(\mu_d, \sigma^2)$,

we reject H_0 when $t = \dfrac{\bar{d} - 0}{S_d / \sqrt{5}} > t_{4,.01} = 3.747$.

$\Sigma d_i = 35 + 9 + 26 - 24 + 17 = 63$, so $\bar{d} = 63/5 = 12.6$

$\Sigma d_i^2 = 1225 + 81 + 676 + 576 + 289 = 2847$,

so $S_d^2 = \left[2847 - (63)^2/5\right]/4 = 513.3$, so $S_d = 22.66$

$t = \dfrac{12.6\sqrt{5}}{22.66} = 1.243 < 3.747$, so we fail to reject $H_0: \mu_d = 0$ at level $\alpha = 0.01$.

2 This is another paired comparison, where the variable of interest is

$$d = [\% \text{ Male (Winter)} - \% \text{ Males (Summer)}].$$

The data are $d_1 = 72-53 = 19$, $d_2 = 47-51 = -4$, $d_3 = 17$, $d_4 = 7$, $d_5 = 9$, $d_6 = 0$, $d_7 = -9$, $d_8 = 10$.

Assume $d_1, d_2, \ldots, d_8 \sim N(\mu_d, \sigma^2)$.

Test $H_0: \mu_d = 0$
vs $H_a: \mu_d > 0$ at level $\alpha = .05$

Reject H_0 if $t = \dfrac{\bar{d} - 0}{S/\sqrt{8}} > t_{7,.05} = 1.895$.

$\Sigma d_i = 19 - 4 + 17 + 7 + 9 + 0 - 9 + 10 = 49$, so $\bar{d} = \frac{49}{8} = 6.125$

$\Sigma d_i^2 = 361 + 16 + 289 + 49 + 81 + 0 + 81 + 100 = 977$,

so $S_d^2 = \dfrac{\Sigma d_i^2 - \frac{(\Sigma d_i)^2}{8}}{7} = [977 - (49)^2/8]/7 = 96.6964$,

so $S_d = 9.833$.

$$t = \frac{6.125\sqrt{8}}{9.833} = 1.762 < 1.895,$$

so we fail to reject $H_0: \mu_d = 0$ at level $\alpha = 0.05$.

3 (a) $SS_x = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}$

$\Sigma x_i = 6.7 + 5.1 + 4.2 + 3.3 + 2.1 = 21.4$

$\Sigma x_i^2 = 44.89 + 26.01 + 17.64 + 10.89 + 4.41 = 103.84$,

so $SS_x = [103.84 - (21.4)^2/5] = \underline{\underline{12.248}}$

$SS_Y = \Sigma(y_i - \bar{y}) = \Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n}$,

$\Sigma y_i = 43.6 + 32.9 + 26.2 + 16.2 + 13.9 = 132.8$

$\Sigma y_i^2 = 1900.96 + 1082.41 + 686.44 + 262.44 + 193.21 = 4125.46$,

so $SS_y = 4125.46 - (132.8)^2/5 = \underline{\underline{598.292}}$

$SS_{xy} = \Sigma(x_i - \bar{x})(y_i - \bar{y}) = \Sigma x_i y_i - (\Sigma x_i)(\Sigma x_i)/n$

$\Sigma x_i y_i = (6.7)(43.6) + (5.1)(32.9) + (4.2)(26.2) + (3.3)(16.2) + (2.1)(13.9)$

$= 292.12 + 167.79 + 110.04 + 53.46 + 29.19$

$= 652.6$,

so $SS_{xy} = 652.6 - (21.4)(132.8)/5 = \underline{\underline{84.216}}$

$$r = \frac{SS_{xy}}{\sqrt{SS_x \cdot SS_y}} = \frac{84.216}{\sqrt{(12.248)(598.292)}} = \frac{84.216}{85.6030} = \underline{\underline{0.9838}}$$

$$b = \frac{SS_{xy}}{SS_x} = \frac{84.216}{12.248} = \underline{\underline{6.876}}$$

$$a = \bar{y} - b\bar{x} = \frac{132.8}{5} - 6.876\,\frac{21.4}{5} = -2.869$$

So $\hat{y} = -2.869 + 6.876x$.

(continued)

(3) (continued)

Using the last formula on the previous page, we obtain:

x_i	6.7	5.1	4.2	3.3	2.1	
$\hat{y}_i = -2.869 + 6.876x_i$	43.200	32.199	26.010	19.822	11.570	
y_i	43.6	32.9	26.2	16.2	13.9	Σ ↓
$y_i - \hat{y}_i$	0.4	0.70	0.19	-3.62	2.33	0
$(y_i - \hat{y}_i)^2$	0.16	0.49	.0361	13.104	5.429	19.219

$$S_e^2 = S^2 = \frac{\Sigma (y_i - \hat{y}_i)^2}{n-2} = \frac{19.219}{3} = 6.4064, \text{ so } S = \underline{\underline{2.532}}$$

(b) $H_0: \beta = 0$
$H_a: \beta > 0$ $\quad \alpha = .01$

Reject H_0 if $t = \dfrac{b-0}{S/\sqrt{\Sigma (x_i - \bar{x})^2}} > t_{3,.01} = 4.541$

$$t = \frac{6.876\sqrt{12.298}}{2.532} = 9.504 > 4.541,$$

so reject $H_0: \beta = 0$ at level $\alpha = .01$, and conclude $\beta > 0$.

Since $t_{3,.005} = 5.841$ and $t_{3,.001} = 10.214$, we have

$$.001 < \text{P-value} < .005.$$

(c) 95% C.I for β is $b \pm t_{3,.025} \dfrac{S}{\sqrt{\Sigma (x_i - \bar{x})^2}}$ OR $6.876 \pm 3.182 \dfrac{2.532}{\sqrt{12.298}}$

OR 6.876 ± 2.302 OR $4.57 < \beta < 9.18$.

1 unit change breathing air corresponds to 6.876 units change breathing oxygen.

4 (a) The calculations are omitted. They are exactly as in problem 3, part (a).

(b) $H_0: \beta = 0$

$H_a: \beta > 0$ $\qquad \alpha = .01$

Reject H_0 if $t = \dfrac{b-0}{s/\sqrt{\Sigma(x_i-\bar{x})^2}} > t_{6-2,\,.01} = 3.747$

$t = \dfrac{0.758\sqrt{1.733}}{0.1527} = 6.534 > 3.747$, so reject H_0 at level $\alpha = .01$ and conclude $\beta > 0$.

Since $t_{4,\,.005} = 4.604$ and $t_{4,\,.001} = 7.173$, we have $.001 < p\text{-value} < .005$.

(c) 90% CI for β is $b \pm t_{4,\,.10}\dfrac{s}{\sqrt{\Sigma(x_i-\bar{x})^2}}$

OR $0.758 \pm 2.132\,\dfrac{0.1527}{\sqrt{1.733}}$

OR $0.758 \pm .2473$ OR $0.510 < \beta < 1.005$.

Each \$1000 in list price corresponds to an increase of dealer price of between \$510 and \$1005.

(5) $H_0: \mu_1 = \mu_2 = \mu_3$

vs H_a: not all μ_i's are equal

$\alpha = .01$

Reject H_0 if $F = \dfrac{SSB/(k-1)}{SSW/(n-k)} > F_{k-1, n-k, .01}$

$k = 3$ and $n = 7+4+6 = 17$, so the critical value

is $F_{2,14,.01} = 6.51 \leftarrow$ (Unfortunately not available in our textbook. This can be found in tables elsewhere, or by using Minitab inverse F-distr.)

$T_1 = 61+34+25+12+79+55+20 = 286$

$T_2 = 25+18+54+67 = 164$

$T_3 = 12+36+69+27+18+14 = 176$

$T = 286+164+176 = 626$

$\sum\sum X_{ij}^2 = 3721+1156+625+144+6241+3025+400$
$+ 625+324+2916+4489$
$+ 144+1296+4761+729+324+196 = 31,116$

$SST = \sum\sum X_{ij}^2 - \dfrac{T^2}{n} = 31,116 - \dfrac{(626)^2}{17} = \underline{8064.470}$

$SSB = \sum \dfrac{T_i^2}{n_i} - \dfrac{T^2}{n} = \dfrac{(286)^2}{7} + \dfrac{(164)^2}{4} + \dfrac{(176)^2}{6} - \dfrac{(626)^2}{17}$

$= 11685.143 + 6724 + 5162.667 - 23051.529 = \underline{520.280}$

so $SSW = SST - SSB = 8064.470 - 520.280 = \underline{7544.190}$

5 (continued)

ANOVA table

Source of Variation	Sum of Squares	d.f.	Mean Square	F = MSB/MSW
Between groups	520.280	2	260.14	0.48
Within groups	7544.190	14	538.87	
Total	8064.470	16	—	

Since $F = \frac{MSB}{MSW} = 0.48 < 6.51$, we fail to reject H_0 at level $\alpha = .01$.

6 $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$
H_a: not all μ_i's are equal
$\alpha = .05$
Here $k = 4$ and $n = 5+4+5+3 = 17$

So Reject H_0 if $F = \frac{MSB}{MSW} > F_{3,13,.05} = 3.41$

$T_1 = 7.3+6.9+7.5+6.8+6.2 = 34.7$
$T_2 = 6.6+7.1+7.7+8.0 = 29.4$
$T_3 = 4.2+5.9+4.9+5.1+4.5 = 24.6$
$T_4 = 4.4+5.1+6.2 = 15.7$

$T = 34.7+29.4+24.6+15.7 = 104.4$

(continued)

6 (continued)

$$\sum\sum x_{ij}^2 = 53.29 + 47.61 + 56.25 + 46.24 + 38.44 + 43.56 + 50.41 + 59.29 + 64 + 17.64 + 34.81 + 24.01 + 26.01 + 20.25 + 19.36 + 26.01 + 38.44 = 665.62$$

$$SST = 665.62 - \frac{(104.4)^2}{17} = 665.62 - 641.139 = \underline{24.481}$$

$$SSB = \frac{(34.7)^2}{5} + \frac{(29.4)^2}{4} + \frac{(24.6)^2}{5} + \frac{(15.7)^2}{3} - 641.139$$

$$= 240.818 + 216.090 + 121.032 + 82.163 - 641.139$$

$$= \underline{18.964}$$

$$SSW = SST - SSB = 24.481 - 18.964 = 5.517$$

$$MSW = SSW/13 = 0.4244$$

$$MSB = SSB/3 = 6.321$$

$$F = MSB/MSW = 6.321/0.4244 = 14.894 > 3.41,$$

so reject H_0 at level $\alpha = .05$ and conclude not all of $\mu_1, \mu_2, \mu_3, \mu_4$ are equal.

ANOVA Table

Source	S.S.	d.f.	M.S.	F
Between	18.964	3	6.321	14.894
Within	5.517	13	0.4244	
Total	24.481	16	—	