Solutions to Assignment #2

5.14 (a) $S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

(b) When there are N possible outcomes, "at random" means: each outcome is equally likely, and hence has prob $1/N$.
So each outcome has prob $1/10$.

(c) Clearly $0 < 1/10 < 1$, so each outcome has probability in $(0,1)$. Also
$$\sum_{i=0}^{9} (1/10) = 10(1/10) = 1. \checkmark$$

5.16 (a) 10-stage experiment with two possible outcomes (T or F) at each stage. By the multiplication principle there are $\underbrace{2 \times 2 \times \cdots \times 2}_{10 \text{ times}} = 2^{10} = 1024$ outcomes.

(b) $\{\text{at least one wrong}\}^c = \{\text{all answers correct}\}$

(c) $P\{\text{at least one wrong}\} = 1 - P\{\text{all correct}\}$

$$= 1 - \frac{1}{1024} = \frac{1023}{1024} \approx 0.99902$$

5.18 (a) $S = \{FF, FM, MF, MM\}$

(b) $P\{\text{both girls}\} = P\{FF\} = \frac{1}{4}$

(c) $P\{FF\} = (.49)(.49) = 0.2401.$

5.22 False. The two trials are clearly *not* indep. The conditional prob of voting Republican in 2004, given ~~that~~ that one voted Republican in 2000, must be close to 1.

5.24

Enviro. Group \ pay higher	YES	NO	
YES	30	66	96
NO	88	933	1021
	118	999	1117

(a) (i) ~~P[Envir~~ P[enviro-group-YES] $= \frac{96}{1117} = 0.0859$

P[pay higher-YES] $= \frac{118}{1117} = 0.1056$

(b) P[(group-YES) $\cap$ (pay higher-YES)] $= 0.0269$

(c) *If* the variables were indep., then the prob. in part (b) would be P[group-YES] × P[pay higher-YES] =

$= (0.0859)(0.1056) = 0.00907$, which is less than .0269

(d) (i) Prob $= (30+66+88)/1117 = 184/1117 = 0.1647$

(ii) P[(enviro-YES) $\cup$ (pay higher-YES)] $= P(A) + P(B) - P(A \cap B)$
A B
$= \frac{96}{1117} + \frac{118}{1117} - \frac{30}{1117} = \frac{184}{1117} = 0.1647$

5.26

(a)

Fall ↓ \ Winter →	Y	N	
Y	0.30	0.10	0.40
N	0.05	0.55	0.60
	0.35	0.65	1.00

(b) $P(F) = P(YY) + P(YN) = 0.30 + 0.10 = 0.40$

$P(W) = P(YY) + P(NY) = 0.30 + 0.05 = 0.35$

(c) $P[F \text{ and } W] = P[\text{bought from both catalogs}] = P(YY) = 0.30$

(d) $P[F \cap W] = 0.30$, but $P(F)\,P(W) = (0.40)(0.35) = 0.14 \neq 0.3$

so F and W are not indep.

One possible explanation for non-independence is that some shoppers may be strictly seasonal: they buy only from the Fall catalogue or only from the Winter catalog.

5.30 (a) $P[\text{Yes} \mid <\$25k] = \dfrac{P[<\$25k \cap \text{Yes}]}{P[<\$25k]} = \dfrac{.0011}{.0011+0.1747}$

$= \dfrac{.0011}{.1758} = 0.006257$

(b) $P[<\$25k \mid \text{Yes}] = \dfrac{P[<\$25k \cap \text{Yes}]}{P[\text{Yes}]} = \dfrac{.0011}{(.0011+.0009+.0009+.0010)}$

$= \dfrac{.0011}{.0039} = 0.2820$

5.34 (a) I am sorry that I didn't teach you about "contingency tables" and related jargon (eg. "cross-tabulate"). What is asked for is to arrange the frequencies in the following table:

	2nd free throw made	2nd free throw missed	
First free throw made	251	34	285
First free throw missed	48	5	53
	299	39	338

(b) $P[\text{made first}] = \dfrac{251+34}{338} = \dfrac{285}{338} = 0.8432$

$P[\text{made } 2^{nd}] = \dfrac{299}{338} = .8846$

(c) $P[\text{made } 2^{nd} \mid \text{made first}] = P[\text{made both}] / P[\text{made first}] = \dfrac{251/338}{285/338}$

$= 251/285 = 0.8807$ — hardly matters at all.

5.40 $P[\text{makes first serve}] = .52$, so $P[\text{faults first serve}] = 1 - .52 = .48$.

$P[\text{makes } 2^{nd} \text{ serve} \mid \text{faults first serve}] = 0.91$,

so $P[\text{faults } 2^{nd} \text{ serve} \mid \text{faults first serve}] = 1 - .91 = 0.09$.

So $P[\text{double fault}] = P[(\text{fault on } 1^{st}) \cap (\text{fault on } 2^{nd})]$

$= P[\text{fault on } 1^{ST}] \cdot P[\text{fault on } 2^{nd} \mid \text{fault on } 1^{st}]$

$\doteq 0.48\ (0.09) = \underline{\underline{0.0432}}$

5.44 This is a standard "sampling without replacement" calculation:

$P[\text{no winning numbers}] = \frac{43}{49}\ \frac{42}{48}\ \frac{41}{47}\ \frac{40}{46}\ \frac{39}{45}\ \frac{38}{44}$

$\doteq 0.43596$

5.46

outcome	events			
HHH	A	B	C	D
HHT	A	B	C	
HTH	A			
HTT	A			
THH		B		
THT		B		
TTH				
TTT				

(a) $P(A) = 4/8 = 1/2$

$P(B) = 4/8 = 1/2$

$P(C) = \frac{2}{8} = \frac{1}{4}$

$P(D) = 1/8$

(continued)

5.46 (continued)

(b) $A \cap B = HHH \cup HHT$, so $P(A \cap B) = \frac{2}{8} = \frac{1}{4} = \frac{1}{2}\cdot\frac{1}{2} = P(A)P(B)$

so A and B are indep.

$A \cap C = C$, so $P(A)P(C) = \frac{1}{2}\cdot\frac{1}{4} \neq \frac{1}{4} = P(C)$, so

A and C are not indep.

Noting that $A \cap D = D$, $B \cap C = C$, $B \cap D = D$, $C \cap D = D$,

a similar calculation shows that none of the other pairs of events are indep.

So $\{A, B\}$ is the only pair of indep. events.

5.48 (a) The probability must be much lower. In the previous problem, there were $\binom{25}{2} = 300$ potential matching pairs. But in the present problem, there are only 24 ways to pair your fixed birthdate with that of another student.

(b) $1 - P[\text{no match of your birthday}] = 1 - \left(\frac{364}{365}\right)^{24}$

$= 1 - 0.9363 = 0.0637$

5.54 $\left(\frac{1}{50}\right)^7 = (0.02)^7 = 1.28 \times 10^{-12}$

5.62 (a) $P(YES) = 54/5228 = 0.0103$

(b) Unfortunately "sensitivity" and "specificity" seem to be undefined, both here and elsewhere in the text. From the answer key that I have, it appears that

$$\text{sensitivity} \stackrel{\text{def}}{=} P[Pos \mid Yes]$$

and

$$\text{specificity} \stackrel{\text{def}}{=} P[NEG \mid NO],$$

both of which we would like to be close to 1.

In this case, $\text{sensitivity} = P[POS \mid YES] = \dfrac{P[POS \cap YES]}{P[YES]}$

$$= \frac{48/5282}{54/5282} = \frac{48}{54} = \frac{8}{9} = \underline{\underline{0.8889}}$$

and $\text{specificity} = P[NEG \mid NO] = \dfrac{P[NEG \cap NO]}{P[NO]}$

$$= \frac{3921/5282}{5228/5282} = \frac{3921}{5228} = \underline{\underline{0.7500}}$$

(c) $P[YES \mid POS] = \dfrac{P[YES \cap POS]}{P[POS]} = \dfrac{48/5282}{1355/5282} = \dfrac{48}{1355} = 0.0\underline{\underline{3542}}$

$P[NO \mid NEG] = \dfrac{P[NO \cap NEG]}{P[NEG]} = \dfrac{3921/5282}{3927/5282} = \dfrac{3921}{3927} = \underline{\underline{0.9984}}$

(continued)

5.62 (d) There are four ways of describing the probability that a diagnostic test makes a correct decision. First, in (b-i), we see the probability that the test would be positive if one had Down Syndrome. Second, in (b-ii), we see the probability that the test would be negative given that one does not have Down Syndrome. In (c-i), we see the probability that one would have Down Syndrome if the test is positive. Finally, in (c-ii), we see the probability that one would not have Down Syndrome given a negative test.

5.70

(a) $(.9)^{12} = 0.28243$

(b) $(.5)^{12} = 0.0002441$

5.76 (a) $P[\text{seen} \cap \text{YES}] = 0.10$ (picked off from the table)

(b) $P[\text{YES} \mid \text{seen}] = \dfrac{P[\text{YES} \cap \text{seen}]}{P[\text{seen}]} = \dfrac{0.10}{0.10+0.25} = \dfrac{10}{35}$

$= 0.2857$

(c) (a) $P(A \cap B)$

(b) $P(A \mid B)$

(d) NO, since $P(A) \neq P(A \mid B)$.

[Here $P(A) = .10 + .05 = .15$ and $P(A \mid B) = 0.2857$.

Equivalently, you could verify that A and B are not indep by checking that either

$$P(B) \neq P(B \mid A)$$

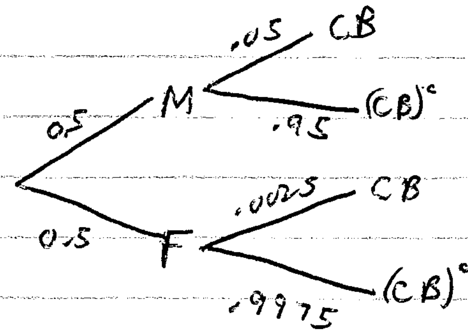
or $P(A \cap B) \neq P(A) \cdot P(B)$.

5.88 Let CB = color blindness, M = male, F = female.

(a) $P(CB|M) = 0.05$, $P(CB|F) = 0.0025$.

(b) $P(M) = P(F) = 0.5$

Using a tree diagram:



$$P(CB) = P(M)\,P(CB|M) + P(F)\,P(CB|F)$$
$$= 0.5\,(0.05) + 0.5\,(0.0025)$$
$$= .025 + .00125 = \underline{\underline{0.02625}}$$

(c) $$P(F|CB) = \frac{P(F \cap CB)}{P(CB)} = \frac{(0.5)(.0025)}{.002625}$$

$$= \frac{.00125}{0.02625} = \underline{\underline{0.04762}}$$

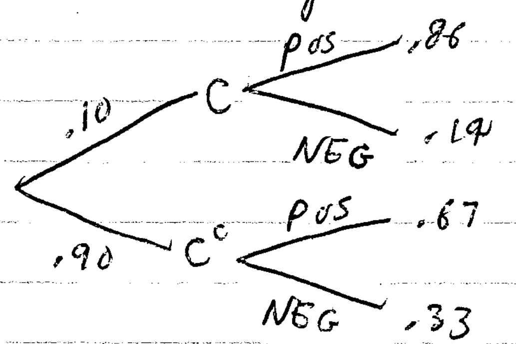
~~(a) (b) Using tree diagram~~ —

5.90 Recall definitions from earlier problem

sensitivity $= P[\text{POS} \mid \text{has cancer}] = 0.86$

specificity $= P[\text{NEG} \mid \text{doesn't have cancer}] = 0.33$

(a)(b) Using tree diagram – Let $C = \{\text{has prostate cancer}\}$



$$P(\text{POS}) = P(C \cap \text{POS}) + P(C^c \cap \text{POS})$$

$$= P(C)\,P(\text{POS} \mid C) + P(C^c)\,P(\text{POS} \mid C^c)$$

$$= .10(.86) + .90(.67) = .086 + .603 = .699$$

So $P(C \mid \text{POS}) = \dfrac{P(C \cap \text{POS})}{P(\text{POS})} = \dfrac{.086}{.699} = 0.1248$

(c) The sensitivity will go up because more people will have positive tests, some of whom actually have prostate cancer. On the other hand more positive tests result in fewer negative ~~tt~~ tests, lowering the specificity.