UNIVERSITY OF CALGARY

DEPARTMENT OF MATHEMATICS AND STATISTICS

STAT 327 — Test no. 1 — February 27, 2009

Marks

NAME: Solutions

10 1. The data values below represent the price per share of five active stocks that were traded yesterday:

19, 6, 2, 23, 14.

For this data set, compute the median, the mean, the variance and the standard deviation.

ordered observations 2, 6, 14, 19, 23

median = 14 ← 3 marks

$\Sigma x_i = 64$, so mean $\bar{X} = \frac{\Sigma x_i}{n} = \frac{64}{5} = 12.8$ ← 3

$\Sigma x_i^2 = 4 + 36 + 196 + 361 + 529 = 1126$

var. $s^2 = \frac{\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}}{n-1} = \frac{1126 - (64)^2/5}{4} = \frac{306.8}{4} = 76.7$ ← 3

std. dev. $s = +\sqrt{76.7} = 8.758$ ← 1

10 2. A study was conducted on the relationship between the number of persons in a household (x) and the number of radios in that household (y). A sample of five households yielded the following data:

no. of persons, x	3	1	5	2	4
no. of radios, y	3	2	6	4	3

7 (a) Find the equation of the least-squares regression line (with y as a function of x) obtained from this data.

x	y	x^2	xy	y^2
3	3	9	9	9
1	2	1	2	4
5	6	25	30	36
2	4	4	8	16
4	3	16	12	9
15	18	55	61	74

$$b = \frac{\sum x_i y_i - \frac{(\sum x_i)(\sum y_i)}{n}}{\sum x_i^2 - (\sum x_i)^2/n} = \frac{61-(15)(18)/5}{55-(15)^2/5}$$

$$= \frac{61-54}{10} = \underline{0.7} \leftarrow 4$$

$$a = \bar{y} - b\bar{x} = 3.6 - 0.7(3)$$

$$= 3.6 - 2.1 = \underline{1.5} \leftarrow 2$$

$$\boxed{\hat{y} = 1.5 + 0.7x} \leftarrow 1$$

3 (b) Compute the correlation, r, between x and y.

$$r = \frac{7}{\sqrt{10}\sqrt{74-\frac{(18)^2}{5}}} = \frac{7}{\sqrt{92}} = \frac{7}{9.5917} = \underline{0.7298} \leftarrow 3$$

10 3. A fair coin is tossed six times. Let X denote the number of times that "Heads" comes up in the six tosses.

3 (a) Find the probability that X is equal to 3.

$$P[X=3] = \binom{6}{3}\left(\frac{1}{2}\right)^3\left(1-\frac{1}{2}\right)^{6-3} = 20\left(\frac{1}{2}\right)^6 = \frac{20}{64} = \frac{5}{16}$$

$$= \underline{\underline{0.3125}} \leftarrow 3$$

3 (b) Find the probability that X is at least 4.

$$P[X \geq 4] = \binom{6}{4}\left(\frac{1}{2}\right)^6 + \binom{6}{5}\left(\frac{1}{2}\right)^6 + \binom{6}{6}\left(\frac{1}{2}\right)^6$$

$$= \left(\frac{1}{2}\right)^6\left[15+6+1\right] = \frac{22}{64} = \frac{11}{32} = \underline{\underline{0.3438}} \leftarrow 3$$

2 (c) Compute the mean of X.

$$EX = np = 6\left(\frac{1}{2}\right) = \underline{\underline{3}} \leftarrow 2$$

[Full credit for getting the right answer the hard way, — that is, by computing $\sum_{x=0}^{6} x \, P[X=x]$.]

2 (d) Compute the standard deviation of X.

$$Var(X) = \sigma^2 = np(1-p) = 6\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 1.5$$

$$\text{So } \sigma = \sqrt{1.5} = \underline{\underline{1.2247}} \leftarrow 2$$

10 4. In a large population, 10% of the people have a certain disease and 90% do not have the disease. When a special diagnostic test is admininistered to an individual, the possible outcomes are POS ("the person appears to have the disease") or NEG ("the person does not appear to have the disease"). It is known that, when the person has the disease, the conditional probability that the diagnostic test yields the result POS is 0.90. Also, it is known that, when the person does not have the disease, the conditional probability that the diagnostic test yields the result POS is 0,20.

5 (a) A person is selected at random from the population and is given the diagnostic test. What is the probability that the outcome is POS?

Let $D = \{$person has the disease$\}$

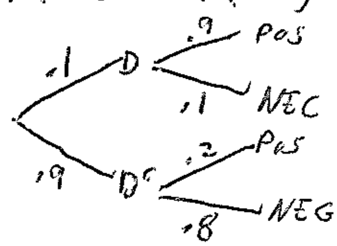
$P(D) = .10$, $P(D^c) = .90$

$P(Pos|D) = 0.90$ $P(Pos|D^c) = 0.20$

$P(Pos) = P(D)P(Pos|D) + P(D^c)P(Pos|D^c)$

$= .1(.9) + .9(.2) = .09 + .18 = \underline{0.27}$ 5

or use tree:



5 (b) Given that the outcome for the randomly-selected person is POS, what is the conditional probability that the person really has the disease?

$$P(D|Pos) = \frac{P(D \cap Pos)}{P(Pos)} = \frac{.09}{.27} = \underline{\frac{1}{3}}$$ 5

10 5. Two fair dice are rolled. Let Y denote the *minimum* of the two numbers that come up. [For example, when the outcome is (3,5), then $Y = 3$; when the outcome is (4,2), then $Y = 2$; and when the outcome is (4,4), then $Y = 4$.]

7 (a) Find the distribution of the random variable Y.

y	$P[Y = y]$
1	11/36
2	9/36
3	7/36
4	5/36
5	3/36
6	1/36
$\Sigma =$	1

(1) mark

(1) mark for each correct prob.

value of min for each outcome

1st die \ 2nd die	1	2	3	4	5	6
1	1	1	1	1	1	1
2	1	2	2	2	2	2
3	1	2	3	3	3	3
4	1	2	3	4	4	4
5	1	2	3	4	5	5
6	1	2	3	4	5	6

3 (b) Compute the expected value of Y.

$$EY = \sum y P[Y=y] = \frac{1}{36}\left[1(11) + 2(9) + 3(7) + 4(5) + 5(3) + 6(1)\right]$$

$$= \frac{1}{36}\left[11 + 18 + 21 + 20 + 15 + 6\right]$$

$$= \frac{91}{36} = \underline{2.5278} \leftarrow (3)$$

50 marks total