STAT 407

# Corrections to Assignment #3 Solutions



Here is the complete solution to Chapter 5, problem 12:

12. (a) $P\{X_1 < X_2 < X_3\}$

$$= P\{X_1 = \min(X_1, X_2, X_3)\}$$

$$P\{X_2 < X_3 | X_1 = \min(X_1, X_2, X_3)\}$$

$$= \frac{\lambda_1}{\lambda_1 + \lambda_2 + \lambda_3} P\{X_2 < X_3 | X_1 = \min(X_1, X_2, X_3)\}$$

$$= \frac{\lambda_1}{\lambda_1 + \lambda_2 + \lambda_3} \frac{\lambda_2}{\lambda_2 + \lambda_3}$$

where the final equality follows by the lack of memory property.

(b) $P\{X_2 < X_3 | X_1 = \max(X_1, X_2, X_3)\}$

$$= \frac{P\{X_2 < X_3 < X_1\}}{P\{X_2 < X_3 < X_1\} + P\{X_3 < X_2 < X_1\}}$$

$$= \frac{\frac{\lambda_2}{\lambda_1 + \lambda_2 + \lambda_3} \frac{\lambda_3}{\lambda_1 + \lambda_3}}{\frac{\lambda_2}{\lambda_1 + \lambda_2 + \lambda_3} \frac{\lambda_3}{\lambda_1 + \lambda_3} + \frac{\lambda_3}{\lambda_1 + \lambda_2 + \lambda_3} \frac{\lambda_2}{\lambda_1 + \lambda_2}}$$

$$= \frac{1/(\lambda_1 + \lambda_3)}{1/(\lambda_1 + \lambda_3) + 1/(\lambda_1 + \lambda_2)}$$

(c) $\frac{1}{\lambda_1 + \lambda_2 + \lambda_3} + \frac{1}{+\lambda_2 + \lambda_3} + \frac{1}{\lambda_3}$

(d) $$\sum_{i \neq j \neq k} \frac{\lambda_i}{\lambda_1 + \lambda_2 + \lambda_3} \frac{\lambda_j}{\lambda_j + \lambda_k} \left[ \frac{1}{\lambda_1 + \lambda_2 + \lambda_3} + \frac{1}{\lambda_j + \lambda_k} + \frac{1}{\lambda_k} \right]$$

where the sum is over all 6 permutations of 1, 2, 3.

Also the "solution" to Chapter 5, problem 25, which was posted, is totally unrelated to the problem. A new solution appears on the next page.

Chapter 5, #25:

(a) The time that you wait for a server to be free is the minimum of 3 indep exponential random variables with rates $\mu_1, \mu_2, \mu_3$ — so the expected waiting time is $\dfrac{1}{\mu_1+\mu_2+\mu_3}$.

Your ~~service~~ expected service time is

$$\sum_{i=1}^{3} P[\text{you get Server } i]\, E[\text{service time} \mid \text{you get Server } i]$$

$$= \sum_{i=1}^{3} \frac{\mu_i}{\mu_1+\mu_2+\mu_3} \cdot \frac{1}{\mu_i} = \sum_{i=1}^{3} \frac{1}{\mu_1+\mu_2+\mu_3} = \frac{3}{\mu_1+\mu_2+\mu_3}.$$

So your expected total time in the system is

$$\frac{1}{\mu_1+\mu_2+\mu_3} + \frac{3}{\mu_1+\mu_2+\mu_3} = \underline{\underline{\frac{4}{\mu_1+\mu_2+\mu_3}}}$$

(b) The expected time from your arrival until service starts for the first customer is $\dfrac{1}{\mu_1+\mu_2+\mu_3}$.

From that point on, it is just as though you had arrived to find no other waiting customer and 3 servers busy, so your expected additional waiting time is the answer to part (a).

So your expected total time is $\dfrac{1}{\mu_1+\mu_2+\mu_3} + \dfrac{4}{\mu_1+\mu_2+\mu_3} = \underline{\underline{\dfrac{5}{\mu_1+\mu_2+\mu_3}}}$.