Statistics 407

E.G. Enns | Assignment 1 | Due: Monday March 8

1) Urns A and B each contain 3 marbles, of which 3 are red, 2 are yellow and 1 is black. At each step of the process, you select one marble at random from each urn and interchange them. The game is over when you have either 3 red marbles or 2 yellow marbles in urn A.
   a) Define the states of the process.
   b) Find the transition matrix in canonical form.
   c) Find N, B, $\underline{\tau}_1$ and $\underline{\tau}_2$.

2) Derive $\underline{\tau}_2$ as defined in class.

3) An urn contains 4 red and 2 white marbles. The marbles are drawn sequentially till none are left. Let N = the number of red marbles drawn before the first white is drawn. Let M = the number of sequences of red marbles.

   a) Find $E(N)$, $E(M)$ and the $cov\ (N,M)$.
   b) Are N and M independent?

4) In the population model discussed in class, we defined $G(z) = E(z^N)$ where N was the number of reproductive offspring of one individual in their lifetime. Let B_n be the number of individuals in the n^th^ generation when $B_0 = 1$. In generation 1,2,… we also have $M_1, M_2, \ldots$ immigrants where the M_i are *i,i,d* random variables with $F(z) = E(z^M)$.
   Let $\theta_n(z) = E\left(z^{B_n}\right)$.

   a) Find an expression for $\theta_n(z)$.
   b) Find $E(B_n)$ and $var\ (B_n)$ in terms of $\mu_1, \mu_2, \sigma_1^2, \sigma_2^2$ where:
   $$E(N) = \mu_1,\ E(M) = \mu_2,\ \text{var}\,N = \sigma_1^2,\ \text{var}\,M = \sigma_2^2$$