or ($ 134.60, $ 1665.40)

Because this interval contains only $^{+}$ve values, we can be quite confident that $\mu_1 - \mu_2 > 0$. Thus, it is reasonable to assume that the mean salary for males exceeds the mean salary for females.

C. I. for $\mu_1 - \mu_2$ when variances are unknown & sample sizes are small (pop. normal)

The problem of finding a C. I. has been solved for the special case when the unknown variances are equal. A general solution has not been found for the case where the unknown variances are unequal.

$\bar{x}_1, \bar{x}_2, \quad s_1^2, \quad s_2^2$

$$S_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$$

100 $(1-\alpha\%)$ C. I $\mu_1 - \mu_2$

C. I.

$$\left(\bar{x}_1 - \bar{x}_2 - t_{\frac{\alpha}{2}, \nu} \cdot \sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}, \ \bar{x}_1 - \bar{x}_2 + t_{\frac{\alpha}{2}, \nu} \cdot \sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}\right)$$

$\nu = n_1 + n_2 - 2$ & S_p^2 is the pooled variance estimate of the common variance!

e.g. Two manufacturing companies produce carbide drills tips that are used to cut holes in steel sheets. A customer wishing to know which drill tips have the longer life. He purchase independent samples of $n_1 = 20$ drill tips from company 1 and $n_2 = 15$ drill tips from company 2. The mean lifes of the drill tips are $\bar{x}_1 = 78$ minutes & $\bar{x}_2 = 84$ minutes. The population variance are unknown but assumed to be equal. The sample variances are $s_1^2 = 41$, $s_2^2 = 36$

Construct a 95% C.I. for $\mu_1 - \mu_2$

Soln:

$$s_p^2 = \frac{(20-1)(41) + (15-1)36}{20+15-2} = \frac{1283}{33} = 38.88$$

95% C.I. $\frac{\alpha}{2} = 0.025$. d.f. $= n_1 + n_2 - 2 = 33$

$$t_{\frac{\alpha}{2}, 33} = 2.04$$

$\therefore$ 95% C.I. for $\mu_1 - \mu_2$,

$$\left(78 - 84 - 2.04\sqrt{\frac{38.88}{20} + \frac{38.88}{15}},\ 78 - 88 + 2.04\sqrt{\frac{38.88}{20} + \frac{38.88}{15}}\right)$$

$$= (-10.34, -1.66)$$

95% C.I. contain only −ve values, $\Rightarrow$ drill tips of company #1 do <u>NOT</u> last as long, on the average by company 2.