subj = 1   1·11 s.d. below the mean
subj = 2   ·63 " " " "
subj 6   1.58 s.d. above the mean

Standard scores are frequently used to obtain comparability of observation by different procedures.

e.g.
1st yr. Calculus exam. scores of 100 students $\mu = 65$, $\sigma = 8$
" " algebra " " of the same 100 stdts $\mu = 52$, $\sigma = 3$

student - tak got Calculus 58, z-score = $\frac{58-65}{8} = -.875$
student - tak got algebra 55, z-score = $\frac{55-52}{3} = 1$

Clearly tak did much poorly in Calculus than Algebra relative to the group of student taking in the exam, although this is NOT reflected in the original marks assigned.

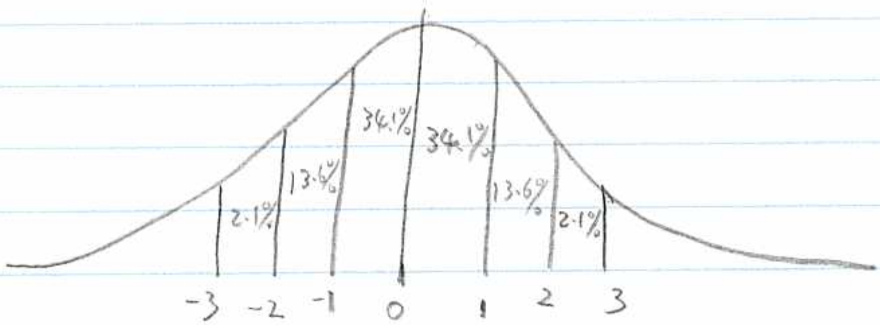
Because z-scores results in -ve, some people prefer to transform them into other distribution.
One distribution that has been widely used is one with a mean of 50 and a s.d. of 10
Such transformed standard score are generally called T-scores. To convert a z-score to a T-score, use $T = 10z + 50$
e.g. with a z-score of 2.5, the T-score would be
$T = 10 \times 2.5 + 50 = 75$
In the new distibution, the mean is 50 and s.d. is 10
a score of 75 is still 2.5 s.d. above the mean

Standard Normal Distribution

A random variable is said to have the standard Normal Distribution if it has the normal distribution with mean $\mu = 0$ and variance $\sigma^2 = 1$ denoted as $N(0, 1)$.

It is common to denote the standard Normal Distribution by Z rather than X.

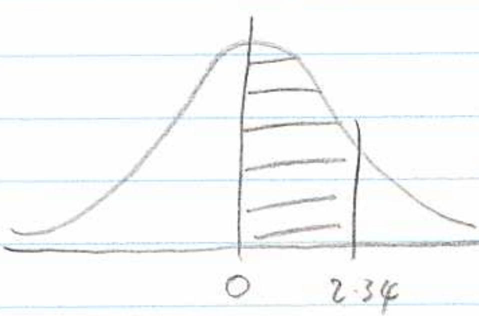
The standard Normal distribution & its area



Calculating Area under the standard Normal Curve

e.g. Area between a mean & a +ve value under the Standard Normal Curve:

Find area under the Standard normal curve between 0 and 2.34. i.e Find $P\{0 \le Z \le 2.34\}$



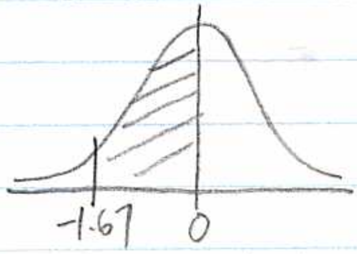
Move down the left-side of the table to the row value of 2.3. Now move across to the hundredth column headed by the digit 4.

The number found at the intersection of this row and the column is .4904 which represents the area between 0 and 2.34. Thus we have $P(0 \le Z \le 2.34) = .4904$

e.g. Area between the mean and a -ve value under the standard Normal curve :

Find area under the standard normal curve between 0 and -1.67

Sol.



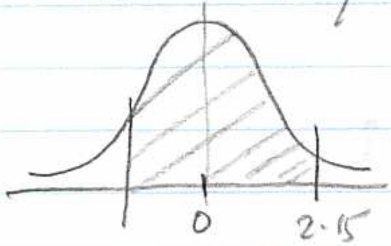
We want to find the shaded area. Because the normal distribution is symmetric, the area between -1.67 & 0 is the same as area between 0 and 1.67.

From table $P(0 \leq Z \leq 1.67) = 0.4525$

$\therefore \quad P(-1.67 \leq Z \leq 0) = 0.4525$.

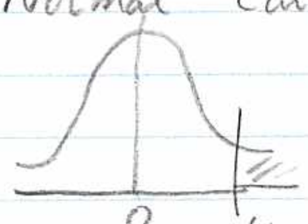
e.g. Area between a -ve and a +ve value under standard Normal Curve $Z \sim N(0,1)$

Find $P\{-1.21 \leq Z \leq 2.15\}$.



$$\begin{aligned} &P\{-1.21 \leq Z \leq 2.15\} \\ &= P\{-1.21 \leq Z \leq 0\} + P\{0 \leq Z \leq 2.15\} \\ &= 0.3869 + 0.4842 = 0.8711 \end{aligned}$$

e.g. Area in the right-hand tail of the standard Normal curve. Find $P(Z > 1.64)$

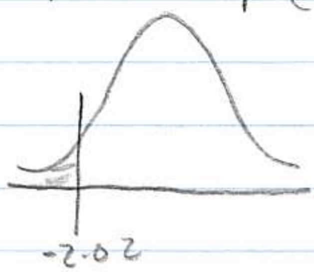


$$\begin{aligned} P(Z > 1.64) &= 0.5 - P(0 \leq Z \leq 1.64) \\ &= 0.5 - 0.4495 = 0.0505 \end{aligned}$$

3.23

e.g. Area in the left hand tail of the standard normal curve.

Find $P(z \le -2.02)$

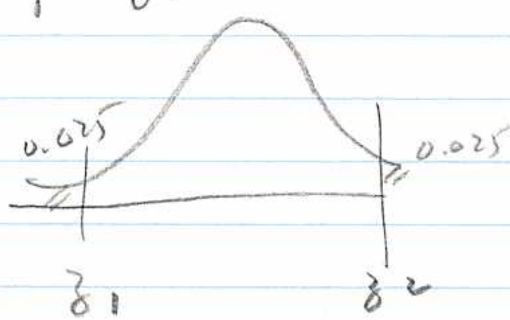


$P(0 \le z \le 2.02)$
$= P(-2.02 \le z \le 0)$
$= 0.4783$

$\therefore P(z \le -2.02) = 0.5 - 0.4783$
$= 0.0217$.

e.g. Find a z-score associated with a specific area

Find the values of z_1 and z_2 such that the area of right of z_2 is 0.025 and area the left of $z_1 = 0.025$



From table,
$z_2 = 1.96$,
$z_1 = -1.96$

e.g. Area under $N(\mu, \sigma^2)$

$X \sim N(10, 25)$

Find $P\{12 \le X \le 16\} = P\left\{\frac{12-\mu}{\sigma} \le \frac{X-\mu}{\sigma} \le \frac{16-\mu}{\sigma}\right\}$

$= P\left\{\frac{12-10}{5} \le z \le \frac{16-10}{5}\right\} = P\{0.4 \le z \le 1.2\}$

$= 0.2295$