is measured on ordinal scale.
The Friedman test involves ranking the scores for each subject across the different conditions.

e.g. H_0: Nurses' scores on aggressiveness of nursing care are unrelated to the type of patients' illness.

Score (rank)

Nurse	AIDS		Cancer		Alzheimer	
1	17	(2)	18	(3)	15	(1)
2	15	(2)	20	(3)	11	(1)
3	14	(3)	12	(1)	13	(2)
4	11	(1)	19	(3)	18	(2)
5	18	(2)	20	(3)	17	(1)
6	16	(3)	14	(1)	15	(2)
7	12	(1)	14	(3)	13	(2)
8	9	(1)	13	(3)	12	(2)
9	16	(2)	17	(3)	15	(1)
	R_1	17	R_2	23	R_3	14

$$\chi^2_{calc} = \left[\frac{12\left(\Sigma R^2\right)}{N\,k(k+1)}\right] - 3N(k+1) \sim \chi^2(k-1)$$

k : # conditions
R : sum of the ranks for each k condition
Σ : sum of squared sum of ranks (R^2) for the k conditions
N : # of subjects

$$\chi^2_{calc} = \frac{12\left(17^2+23^2+14^2\right)}{9\,(3)\,(4)} - (3)(9)(4) = 4.67$$

$\chi^2_{0.05}(2) = 5.99$, $\chi^2_{calc} = 4.67 < 5.99$ DO NOT reject H_0

```
get file='d:\stat601.14\polit\p212.sav'.
npar tests friedman=aids cancer alzh.
```

7.14

NPar Tests:p212.spo

Friedman Test

Ranks

	Mean Rank
aids	1.89
cancer	2.56
alzh	1.56

= 17/9
= 23/9
= 14/9

Test Statistics[a]

N	9
Chi-Square	4.667
df	2
Asymp. Sig.	.097

a. Friedman Test

non parametric approach

```
GLM
  aids cancer alzh
  /WSFACTOR = disease 3 Polynomial
  /METHOD = SSTYPE(3)
  /EMMEANS = TABLES(disease)
  /PRINT = DESCRIPTIVE
  /CRITERIA = ALPHA(.05)
  /WSDESIGN = disease .
```

parametric approach

General Linear Model

Within-Subjects Factors

Measure: MEASURE_1

disease	Dependent Variable
1	aids
2	cancer
3	alzh

Descriptive Statistics

	Mean	Std. Deviation	N
aids	14.2222	2.99073	9
cancer	16.3333	3.12250	9
alzh	14.3333	2.29129	9

7.14a

Multivariate Tests[b]

Effect		Value	F	Hypothesis df	Error df	Sig.
disease	Pillai's Trace	.402	2.349[a]	2.000	7.000	.166
	Wilks' Lambda	.598	2.349[a]	2.000	7.000	.166
	Hotelling's Trace	.671	2.349[a]	2.000	7.000	.166
	Roy's Largest Root	.671	2.349[a]	2.000	7.000	.166

a. Exact statistic

b.
Design: Intercept
Within Subjects Design: disease

Mauchly's Test of Sphericity[b]

Measure: MEASURE_1

Within Subjects Effect	Mauchly's W	Approx. Chi-Square	df	Sig.
disease	.992	.059	2	.971

Tests the null hypothesis that the error covariance matrix of the orthonormalized transformed dependent variables is proportional to an identity matrix.

Mauchly's Test of Sphericity[b]

Measure: MEASURE_1

Within Subjects Effect	Epsilon[a]		
	Greenhouse-Geisser	Huynh-Feldt	Lower-bound
disease	.992	1.000	.500

Tests the null hypothesis that the error covariance matrix of the orthonormalized transformed dependent variables is proportional to an identity matrix.

a. May be used to adjust the degrees of freedom for the averaged tests of significance. Corrected tests are displayed in the Tests of Within-Subjects Effects table.

b.
Design: Intercept
Within Subjects Design: disease

Tests of Within-Subjects Effects

Measure: MEASURE_1

Source		Type III Sum of Squares	df	Mean Square	F	Sig.
disease	Sphericity Assumed	25.407	2	12.704	2.564	.108
	Greenhouse-Geisser	25.407	1.983	12.810	2.564	.109
	Huynh-Feldt	25.407	2.000	12.704	2.564	.108
	Lower-bound	25.407	1.000	25.407	2.564	.148
Error(disease)	Sphericity Assumed	79.259	16	4.954		
	Greenhouse-Geisser	79.259	15.867	4.995		
	Huynh-Feldt	79.259	16.000	4.954		
	Lower-bound	79.259	8.000	9.907		

7.15

Tests of Within-Subjects Contrasts

Measure: MEASURE_1

Source	disease	Type III Sum of Squares	df	Mean Square	F	Sig.
disease	Linear	.056	1	.056	.011	.920
	Quadratic	25.352	1	25.352	5.363	.049
Error(disease)	Linear	41.444	8	5.181		
	Quadratic	37.815	8	4.727		

Tests of Between-Subjects Effects

Measure: MEASURE_1
Transformed Variable: Average

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Intercept	6045.037	1	6045.037	430.649	.000
Error	112.296	8	14.037		

Estimated Marginal Means

disease

Measure: MEASURE_1

disease	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
1	14.222	.997	11.923	16.521
2	16.333	1.041	13.933	18.733
3	14.333	.764	12.572	16.095

Mann-Whitney U-Test

The Mann-Whitney U-test is a popular nonparametric analog of the independent groups t-test. This statistic tests the null hypothesis that two population distributions are identical against the alternative hypothesis that the distributions are NOT identical.

Suppose we had 2 groups of burn patient (Group A and Group B) who obtained the following scores on a scale measuring positive body image:

Group A: 14, 19, 11, 22, 17

Group B: 10, 16, 15, 18, 13

Score	Group	Rank (A)	Rank (B)
10	B		1
11	A	2	
13	B		3
14	A	4.	
15	B		5
16	B		6
17	A	7	
18	B		8
19	A	9	
22	A	10	
		$R_A = 32$	$R_B = 23$

$$U_A = n_A n_B + \frac{n_A(n_A+1)}{2} - R_A$$

$$U_B = n_A n_B + \frac{n_B(n_B+1)}{2} - R_B$$

where n_A : # of obs. of group A
n_B : # of obs. of group B
R_A : sum of ranks for group A
R_B : sum of ranks for group B

$U = \min(U_A, U_B)$
$U_A = 8$, $U_B = 17$

$U = \min(8, 17) = 8$

Look up critical values ($\alpha = 0.05$) of U statistics table B-5 on Page 476

if $U < U_{0.05}(n_1, n_2)$ Reject H_0

$U_{0.05}(5, 5) = 2$

$U = 8 > 2$ Do <u>NOT</u> reject H_0 at 0.05 level.

Note that table B-5 is appropriate only when sample size of both groups is 20 or less.

When n for either group is > 20 the value of U approaches a normal distribution.

$$z = \frac{U - n_A n_B / 2}{\sqrt{\frac{n_A n_B (n_A + n_B + 1)}{12}}}$$

7.17a

TABLE B–5. CRITICAL VALUES OF THE *U* STATISTIC FOR α = .05 (TWO-TAILED TEST)

$n_1 \rightarrow$ / $n_2 \downarrow$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	—[a]	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
2	—	—	—	—	—	—	—	0	0	0	0	1	1	1	1	1	2	2	2	2
3	—	—	—	—	0	1	1	2	2	3	3	4	4	5	5	6	6	7	7	8
4	—	—	—	0	1	2	3	4	4	5	6	7	8	9	10	11	11	12	13	13
5	—	—	0	1	2	3	5	6	7	8	9	11	12	13	14	15	17	18	19	20
6	—	—	1	2	3	5	6	8	10	11	13	14	16	17	19	21	22	24	25	27
7	—	—	1	3	5	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34
8	—	0	2	4	6	8	10	13	15	17	19	22	24	26	29	31	34	36	38	41
9	—	0	2	4	7	10	12	15	17	20	23	26	28	31	34	37	39	42	45	48
10	—	0	3	5	8	11	14	17	20	23	26	29	33	36	39	42	45	48	52	55
11	—	0	3	6	9	13	16	19	23	26	30	33	37	40	44	47	51	55	58	62
12	—	1	4	7	11	14	18	22	26	29	33	37	41	45	49	53	57	61	65	69
13	—	1	4	8	12	16	20	24	28	33	37	41	45	50	54	59	63	67	72	76
14	—	1	5	9	13	17	22	26	31	36	40	45	50	55	59	64	67	74	78	83
15	—	1	5	10	14	19	24	29	34	39	44	49	54	59	64	70	75	80	85	90
16	—	1	6	11	15	21	26	31	37	42	47	53	59	64	70	75	81	86	92	98
17	—	2	6	11	17	22	28	34	39	45	51	57	63	67	75	81	87	93	99	105
18	—	2	7	12	18	24	30	36	42	48	55	61	67	74	80	86	93	99	106	112
19	—	2	7	13	19	25	32	38	45	52	58	65	72	78	85	92	99	106	113	119
20	—	2	8	13	20	27	34	41	48	55	62	69	76	83	90	98	105	112	119	127

Note: To be statistically significant, the calculated *U* must be *equal to or less than* the tabled value.
[a]A dash indicates that no decision is possible for the specified *n*s.

sig. if U_{calc} < the tabled value

7/18

This is from Polit's Page 203 (Mann-Whitney U-Test).

```
-> get file='d:\stat601.14\polit\p203.sav'.

-> npar test m-w=score by gp(1,2).
```

- - - - - Mann-Whitney U - Wilcoxon Rank Sum W Test

$$Z = \frac{8 - \frac{5\times5}{2}}{\sqrt{\frac{25}{12}(11)}} = \frac{-4.5}{\sqrt{22.9167}} = \frac{-4.5}{4.787} = -0.9400$$

SCORE
by GP

Mean Rank	Sum of Ranks	Cases		
6.40	32.00	5	GP	= 1.00
4.60	23.00	5	GP	= 2.00
		--		
		10	Total	

U	W	Exact** 2*(One-Tailed P)	Z	2-Tailed P
8.0	23.0	.4206	-.9400	.3472

**This exact p-value is not corrected for ties.

```
-> t-test groups=gp(1,2)/variables=score.
```

t-tests for Independent Samples of GP

Variable	Number of Cases	Mean	SD	SE of Mean
SCORE				
GP 1	5	16.6000	4.278	1.913
GP 2	5	14.4000	3.050	1.364

Mean Difference = 2.2000

Levene's Test for Equality of Variances: F= .621 P= .453

t-test for Equality of Means

Variances	t-value	df	2-Tail Sig	SE of Diff	95% CI for Diff
Equal	.94	8	.376	2.349	(-3.218, 7.618)
Unequal	.94	7.23	.379	2.349	(-3.320, 7.720)

The Kruskal-Wallis Test (Polit p205)

The Kruskal-Wallis test is the nonparametric counterpart of the simple oneway ANOVA. It is used to analyze the relationship between a dependent variable that is ordinal in nature and a categorical independent variable that has three or more levels. The K-W procedure tests the null hypothesis that the population distributions for the three (or more) independent groups are identical against that there are differences in the distribution. This test should be used only if there are five or more cases per group.

Suppose that we compared the life satisfaction of patients in three nursing homes, using a six-item scale, and obtained the following scores:

Home A : 6, 12, 18, 14, 17
Home B : 15, 19, 16, 20, 10
Home C : 30, 27, 24, 25, 22

Score :	6	10	12	14	15	16	17	18	19	20	22	24	25	27	30
Group :	A	B	A	A	B	B	A	A	B	B	C	C	C	C	C
Rank :	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

n_A : $1+3+4+7+8=23$ → mean rank 4.6
R_B : $2+5+6+9+10=32$ " 6.4
R_C : $11+12+13+14+15=65$ " 13.0

Kruskal and Wallis proposed the following formula for the test statistic, the H statistic:

$$H = \left[\frac{12}{N(N+1)}\right]\left[\sum_{i=1}^{k}\frac{R_i^2}{n_i}\right] - 3(N+1)$$

where N : total sample size

R_i: summed ranks for group i

n_i: # of observations for group i

$$H_{calc} = \left[\frac{12}{15\,(16)}\right]\left[\frac{23^2}{5} + \frac{32^2}{5} + \frac{65^2}{5}\right] - 3 \times 16$$

$$= 0.05\ (105.8 + 204.8 + 845.0) - 48 = 9.78$$

The H statistic has a sampling distribution that approximates a Chi-square distribution with k-1 degrees of freedom, where k is the number of groups.

$H_{calc} = 9.78 > \chi^2_{.05}(2) = 5.99$

Reject the null hypothesis that the distribution of life satisfaction scores in the three nursing homes is identical.

7.21

```
get file='d:\stat601.14\polit\p205.sav'.
get file='d:\stat601.14\polit\p205.sav'.
npar tests k-w=satisfy by home(1,3).
```

NPar Tests:p205.spo

Kruskal-Wallis Test

Ranks

	home	N	Mean Rank
satisfy	1.00 A	5	4.60
	2.00 B	5	6.40
	3.00 C	5	13.00
	Total	15	

Test Statistics[a,b]

	satisfy
Chi-Square	9.780
df	2
Asymp. Sig.	.008

a. Kruskal Wallis Test

b. Grouping Variable: home

Nonparametric approach

```
ONEWAY
  satisfy BY home
  /STATISTICS DESCRIPTIVES HOMOGENEITY
  /MISSING ANALYSIS
  /POSTHOC = SNK TUKEY SCHEFFE LSD ALPHA(.05).
```

parametric approach.

Oneway

Descriptives

satisfy

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean	
					Lower Bound	Upper Bound
1.00 A	5	13.4000	4.77493	2.13542	7.4711	19.3289
2.00 B	5	16.0000	3.93700	1.76068	11.1116	20.8884
3.00 C	5	25.6000	3.04959	1.36382	21.8134	29.3866
Total	15	18.3333	6.56470	1.69500	14.6979	21.9687

Descriptives

satisfy

	Minimum	Maximum
1.00 A	6.00	18.00
2.00 B	10.00	20.00
3.00 C	22.00	30.00
Total	6.00	30.00

Test of Homogeneity of Variances

satisfy

Levene Statistic	df1	df2	Sig.
.351	2	12	.711

ANOVA

satisfy

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	412.933	2	206.467	13.013	.001
Within Groups	190.400	12	15.867		
Total	603.333	14			

Post Hoc Tests

Multiple Comparisons

Dependent Variable: satisfy

	(I) home	(J) home	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	Upper Bound
Tukey HSD	1.00 A	2.00 B	-2.60000	2.51926	.572	-9.3210	4.1210
		3.00 C	-12.20000*	2.51926	.001	-18.9210	-5.4790
	2.00 B	1.00 A	2.60000	2.51926	.572	-4.1210	9.3210
		3.00 C	-9.60000*	2.51926	.006	-16.3210	-2.8790
	3.00 C	1.00 A	12.20000*	2.51926	.001	5.4790	18.9210
		2.00 B	9.60000*	2.51926	.006	2.8790	16.3210
Scheffe	1.00 A	2.00 B	-2.60000	2.51926	.600	-9.6226	4.4226
		3.00 C	-12.20000*	2.51926	.002	-19.2226	-5.1774
	2.00 B	1.00 A	2.60000	2.51926	.600	-4.4226	9.6226
		3.00 C	-9.60000*	2.51926	.009	-16.6226	-2.5774
	3.00 C	1.00 A	12.20000*	2.51926	.002	5.1774	19.2226
		2.00 B	9.60000*	2.51926	.009	2.5774	16.6226
LSD	1.00 A	2.00 B	-2.60000	2.51926	.322	-8.0890	2.8890
		3.00 C	-12.20000*	2.51926	.000	-17.6890	-6.7110
	2.00 B	1.00 A	2.60000	2.51926	.322	-2.8890	8.0890
		3.00 C	-9.60000*	2.51926	.002	-15.0890	-4.1110
	3.00 C	1.00 A	12.20000*	2.51926	.000	6.7110	17.6890
		2.00 B	9.60000*	2.51926	.002	4.1110	15.0890

*. The mean difference is significant at the .05 level.

Homogeneous Subsets

satisfy

	home	N	Subset for alpha = .05 1	2
Student-Newman-Keuls[a]	1.00 A	5	13.4000	
	2.00 B	5	16.0000	
	3.00 C	5		25.6000
	Sig.		.322	1.000
Tukey HSD[a]	1.00 A	5	13.4000	
	2.00 B	5	16.0000	
	3.00 C	5		25.6000
	Sig.		.572	1.000
Scheffe[a]	1.00 A	5	13.4000	
	2.00 B	5	16.0000	
	3.00 C	5		25.6000
	Sig.		.600	1.000

Means for groups in homogeneous subsets are displayed.

a. Uses Harmonic Mean Sample Size = 5.000.