STAT 723

<u>Correction</u> to the solution of Problem 8.6(c).

On page 2 of the <u>Solutions to Assign. #1</u>, the statement "So the null distribution of $F = \frac{1}{1+\frac{m}{n}T}$ is $F_{m,n}$, ..." is incorrect. What I meant to say was the following:

Write $F = \frac{\sum Y_i/m}{\sum X_i/n}$. Then $T = \frac{1}{1+\frac{m}{n}F}$, where the null distribution of F is $F_{m,n}$. To obtain the null distribution of T, note that $t = \frac{1}{1+\frac{m}{n}f}$ is a strictly-decreasing maps of $f \in (0,\infty)$ onto $t \in (0,1)$. So for $t \in (0,1)$, the distr. function at t is

$$G(t) = P[T \le t] = P\left[\frac{1}{1+\frac{m}{n}F} \le t\right] = P\left[t + \frac{m}{n}F \cdot t \ge 1\right]$$

$$= P\left[F \ge \frac{1-t}{t}\,\frac{n}{m}\right] = 1 - F_{m,n}\left(\frac{1-t}{t}\,\frac{n}{m}\right).$$

So, for $t \in (0,1)$, the density function of T is

$$g(t) = G'(t) = -f_{m,n}\left(\frac{1-t}{t}\,\frac{n}{m}\right)\frac{d}{dt}\left[\frac{1-t}{t}\right]\frac{n}{m}$$

$$= -f_{m,n}\left(\frac{1-t}{t}\,\frac{n}{m}\right)\left(-\frac{1}{t^2}\right)\frac{n}{m} = \frac{n}{m t^2}\, f_{m,n}\left(\frac{1-t}{t}\,\frac{n}{m}\right),$$

where $f_{m,n}$ is the density of $F_{m,n}$.