

# Károly Bezdek

Professor - Canada Research Chair (Tier 1) in Computational and Discrete Geometry  
University of Calgary - Department of Mathematics and Statistics  
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## 1 Education

1978: M.Sc. (Diploma) in Mathematics, Eötvös Loránd University, Budapest, Hungary  
1980: Ph.D. (Dr.rer.nat.) in Mathematics, Eötvös Loránd University, Budapest, Hungary  
1985: C.Sc. (Candidate of Science) in Mathematics, Hungarian Academy of Sciences, Budapest, Hungary  
1995: D.Sc. (Doctor of Science) in Mathematics, Hungarian Academy of Sciences, Budapest, Hungary  
1997: Dr.habil. (Habilitation) in Mathematics, Eötvös Loránd University, Budapest, Hungary

## 2 Academic Work

### 2.1 Eötvös Loránd University (ELTE), Budapest, Hungary: 1978-2002

Bezdek has been a faculty member of the Department of Geometry at ELTE between 1978-2003. In particular, he has worked there as Research Assistant (Aug., 1978 - July, 1980), Teaching Assistant (Aug., 1980 - Dec., 1982), Assistant Professor (Jan., 1983 - June, 1989), Associate Professor (July, 1989 - June, 1998), **Széchenyi Professor** (Jan., 1998 - Jan., 2002: a distinguished professorship awarded by the Hungarian government), and **Full Professor** (July, 1998 - June, 2003). Last but not least, between Oct., 1999 - June, 2003 he has been the **Chair** of the Department of Geometry at ELTE. In chronological order, he was the **7th chair** of the Department of Geometry with a list of chairs including a number of world class mathematicians such as László Fejes Tóth (convex and discrete geometry), György Hajós (geometry of numbers, foundation of geometry), and Béla Kerékjártó (topology, foundation of geometry, projective geometry). Bezdek has **served on** a number of decision making **scientific committees**, including the Mathematical Board of the Hungarian Academy of Sciences, the Science Committee of the Hungarian Ministry of Education, and the Mathematical Committee of the Hungarian National Science Foundation. During the period of 1978 - 2003, while being on a number of special leaves from ELTE, Bezdek has held several **visiting professor positions** at research institutions in **Canada, Germany, the Netherlands, and United States**. This included a period of about **7 years at the Department of Mathematics of Cornell University** in Ithaca, New York.

### 2.2 The University of Calgary (U of C), Calgary, Canada: 2003 -

From July, 2003 **Bezdek is a professor of mathematics and Canada Research Chair (Tier 1) of computational and discrete geometry** at the Department of Mathematics and Statistics of the University of Calgary and is the **director** of the Center for Computational and Discrete Geometry (CCDG) at the University of Calgary (U of C). He is one of the longest serving Canada Research Chairs in Canada having a third seven-year term. Through the CCDG, Bezdek has **organized the PIMS Distinguished Chair Lecture Series** between 2006-2008 which attracted numerous distinguished speakers including Henry Cohn (Microsoft, Theory Group, Redmond, USA), Robert Connelly (Cornell University, Ithaca NY, USA), Thomas C. Hales (University of Pittsburgh, USA), Frank Morgan (Williams College, USA), and Rolf Schneider (Albert-Ludwigs University, Freiburg, Germany). The **Fejes Tóth lectures series, started by Bezdek** in 2009, brought to U of C a number of prominent mathematicians, whose visits generated important joint research works. The following is a selection of recent Fejes Tóth speakers whose lectures attracted significant public interest as well: Endre Boros (Rutgers University, NJ, USA), János Pach

(EPFL, Lausanne, Switzerland), Sinai Robins (University of Sao Paulo, Brazil), Marjorie Senechal (Smith College, Northampton, MA, USA), Domokos Szász (Alfréd Rényi Mathematical Institute of the Hungarian Academy of Sciences, Budapest, Hungary), and Elisabeth Werner (Case Western Reserve University, Cleveland, Ohio). The refereed e-journal **Contributions to Discrete Mathematics (CDM)** is dedicated to publishing significant results in a number of areas of pure and applied mathematics. **Bezdek has started CDM** in 2006 and since then he has been serving as an Editor-in-Chief of CDM. Currently he, Michael Drmota (TU Vienna, Austria) and Hugh Williams (U of C) are Editor-in-Chiefs of CDM. CDM is based on the main campus of U of C and it is free for both readers and authors, edited and published online. Between 2004-2007 Bezdek was a **member** of the Scientific Advisory Board, as well as of the Program Committee of the **Banff International Research Station (BIRS)** in Banff. Since 2003 Bezdek has been a **co-organizer of 8 international conferences at BIRS**. This includes the BIRS-CMO 5-day workshop from 2019, which was centered around modelling the geometry of condensed matter. This is a challenging fundamental problem of crystallography and discrete geometry. The meeting generated a number of collaborations focusing on quasi-crystals, which are crystal-like structures discovered by Dan Shechtman (Nobel Prize-2011). Between July - December, 2011, Bezdek was a program and scientific committee **co-chair of the 6 month thematic program**, on discrete geometry and its applications, **at the Fields Institute for Research in Mathematical Sciences in Toronto**. This has been the largest ever international scientific event of discrete geometry, that brought to Toronto more than 400 researchers including many prominent mathematicians. The Distinguished Lecture Series was given by Erik Demaine (MIT, USA; Presburger Award-2013, Nerode Prize-2015), the Coxeter Lecture Series by Stephen Smale (Univ. of California, Berkeley, USA; Fields Medal-1966, Wolf Prize-2007), and the Fejes Tóth Lecture Series by Thomas C. Hales (Univ. of Pittsburgh, USA; Chauvenet Prize-2003, Fulkerson Prize-2009). Bezdek became a **Titular Professor** of the University of Pannonia in Veszprém, Hungary in 2010 and an **Exterior Fellow** of the Alfréd Rényi Institute of Mathematics of the Hungarian Academy of Sciences in Budapest, Hungary in 2016.

### 3 Recent Awards - Honors

*February 13-15, 2015:* The international workshop **"Combinatorial and Convex Geometry Fest"** hosted by the Banff International Research Station, Banff, Canada celebrated the 60th birthdays of Károly Bezdek (University of Calgary, Canada) and Egon Schulte (Northeastern University, USA) with emphases on the combinatorial and convexity aspects of their research works.

*June 19, 2015:* The **"2015 Fejes Tóth Prize"** was awarded by the University of Pannonia, Veszprém, Hungary to Károly Bezdek (University of Calgary, Canada) "in recognition of exceptional creativity in and outstanding contributions to geometry and in recognition of highly successful research collaborations with a number of academic research groups in Hungary". The prize is named after the Hungarian mathematician László Fejes Tóth who was the chair of the Dept. of Mathematics at the Univ. of Pannonia for over 15 years and who is widely regarded as one of the founding fathers of the field discrete geometry.

*June 29 - July 3, 2015:* The **"Geometry and Symmetry (GeoSym 2015)"** conference in honour of Károly Bezdek (University of Calgary, Canada) and Egon Schulte (Northeastern University, USA) on the occasion of their 60th birthdays was hosted by the University of Pannonia, Veszprém, Hungary and "celebrated the many facets of geometry and symmetry influenced by the research works of Károly Bezdek and Egon Schulte".

*Aug 3, 2016:* The **"2016 Great Supervisor Award"** was awarded to Károly Bezdek (University of Calgary, Canada) by the Faculty of Graduate Studies of the University of Calgary "for serving as a great supervisor and outstanding mentor to his graduate students".

*May 15, 2017:* The **"2017 Life-Time Achievement Research Excellence Award"** of the Faculty of Science of the University of Calgary was awarded to Károly Bezdek (University of Calgary, Canada) "in recognition of Károly Bezdek's exceptional and highly productive research work in convex and discrete geometry".

*July 1, 2018:* The **"Springer Festschrift"** published as "Discrete Geometry and Symmetry, Editors: M. Conder, A. Deza, and A. Ivic-Weiss, Springer Proceedings in Mathematics and Statistics, Volume 234, Springer, New York" has been dedicated to the broad aspects of the scientific works of Károly Bezdek (University of Calgary, Canada) and Egon Schulte (Northeastern University, USA) on the occasion of their 60th birthdays.

*October 22, 2020:* The **"2020 Life-Time Achievement Award"** of the Immigrant Services Calgary was awarded to Károly Bezdek (University of Calgary, Canada) for "his accomplishments that range from being one of the longest serving Canada Research Chairs in Canada, solving one of the best-known open questions of computational and discrete geometry in more than 40 years, creating the free e-journal Contributions to Discrete Mathematics, to his work with BIRS in Banff and the Fields Institute for Research in Mathematical Sciences in Toronto."

## 4 Research Work

Bezdek's main field of research is **convex and discrete geometry**, which is a rapidly developing discipline on the boundary of mathematics and computer science. It enables researchers to tackle important problems in areas like robotics, computer graphics, pattern recognition, crystals and quasicrystals, and manufacturing processes. **He has over 140 research publications including 4 books.** "zbMATH" (<https://zbmath.org>) lists **more than 40 co-authors** of Bezdek. He has **co-organized more than 40 international conferences** and has been an **invited speaker at more than 250** international conferences, workshops, and seminars of leading international scientific organizations, conference centers, and academic institutions. **Bezdek's research monographs**

[105] K. Bezdek, *Classical Topics in Discrete Geometry*, **CMS Books in Mathematics, Springer**, New York, 2010 (163 pages);

[115] K. Bezdek, A. Deza, and Y. Ye (Editors), *Discrete Geometry and Optimization*, **Fields Institute Communications**, Volume 69, **Springer**, New York, 2013 (346 pages);

[116] K. Bezdek, *Lectures on Sphere Arrangements - the Discrete Geometric Side*, **Fields Institute Monographs**, Volume 32, **Springer**, New York, 2013 (198 pages);

[131] K. Bezdek and Zs. Lángi, *Volumetric Discrete Geometry*, *Discrete Mathematics and Its Applications*, **Chapman and Hall - CRC Press**, Boca Raton, FL, 2019 (286 pages);

lead the reader to the frontiers of convex and discrete geometry and can be used also as textbooks for graduate level courses. The following **most cited results** of Bezdek **reflect the fruitful interplay** between geometry, linear algebra, geometric analysis, and combinatorics.

(1) *Proving the Planar Kneser-Poulsen Conjecture (1955)*: The Kneser-Poulsen Conjecture states that the volume of the union of finitely many congruent balls (resp., the volume of the intersection of finitely many congruent balls) under arbitrary contraction of the center points does not increase (resp., does not decrease) in Euclidean  $d$ -space for all  $d > 1$ . [80] gives a proof of the Kneser-Poulsen Conjecture in the Euclidean plane, which was one of the best-known open problems of computational and discrete geometry in more than 40 years. In fact, it proves more by showing that the above statement holds for unions (resp., intersections) of non-congruent circular disks as well. Although most parts of the proof presented in [80] work in Euclidean  $d$ -space for  $d > 1$ , when those parts are combined one obtains a proof of the Kneser-Poulsen Conjecture for  $d = 2$  and leaves it open for  $d > 2$ .

[80] K. Bezdek and R. Connelly, *Pushing disks apart - the Kneser-Poulsen conjecture in the plane*, *Journal für die reine und angewandte Mathematik* Volume 553 (2002), Pages 221-236.

(2) *Towards a Theory of Ball-Polyhedra*: [97] and [110] give an extension of the theory of convex polytopes to the family of intersections of finitely many congruent balls, called ball-polyhedra by proving analogues of many classical theorems of convex polytopes for ball-polyhedra in Euclidean  $d$ -space.

[97] K. Bezdek, Zs. Lángi, M. Naszódi and P. Papez, *Ball-polyhedra*, *Discrete and Computational Geometry*, Volume 38, Issue 2 (2007), Pages 201-230.

[110] K. Bezdek and M. Naszódi, *Rigid ball-polyhedra in Euclidean 3-space*, *Discrete and Computational Geometry*, Volume 49, Issue 2 (2013), Pages 189-199.

(3) *Progressing the Foam Problem of Unit Ball Packings in Euclidean Spaces*: The problem is to find the unit ball packing in Euclidean  $d$ -space whose Voronoi cells have the smallest average surface area. This is a relative of Kelvin's foam problem and it is closely related to the densest sphere packing problem as well. [79] (see also [74]) gives the current best lower bound for the surface area of Voronoi cells of unit ball packings in Euclidean  $d$ -space and improves the Rogers upper bound (1958) for the density of unit ball packings in Euclidean  $d$ -space for all  $d \geq 8$ .

[74] K. Bezdek, *On a stronger form of Rogers's lemma and the minimum surface area of Voronoi cells in unit ball packings*, *Journal für die reine und angewandte Mathematik*, Volume 518 (2000), Pages 131-143.

[79] K. Bezdek, *Improving Rogers' upper bound for the density of unit ball packings via estimating the surface area of Voronoi cells from below in Euclidean  $d$ -space for all  $d \geq 8$* , *Discrete and Computational Geometry*, Volume 28 (2002), Pages 75-106.

(4) *Progressing the Boltyanski-Hadwiger Illumination Conjecture (1960)*: The main result of [37] is the following theorem. For every convex  $d$ -polytope with affine symmetry there exist  $8(d-3)$ -dimensional affine subspaces that lie in its exterior and illuminate it, where  $d \geq 3$ . For  $d = 3$  this proves the Boltyanski-Hadwiger Illumination Conjecture for convex polyhedra with affine symmetry. Recall that this longstanding conjecture of convex and discrete geometry states that every convex body can be illuminated by  $2^d$  exterior points in Euclidean  $d$ -space for all  $d > 1$ . Despite many attempts it remains open for  $d \geq 3$ . Thus, partial results carry particular value such as the following one. A subset of the  $d$ -dimensional Euclidean space having nonempty interior is called a spindle convex body if it is the intersection of (finitely or infinitely many) congruent  $d$ -dimensional closed balls. The spindle convex body is called a wide one, if it contains the centers of its generating balls. The main result of [107] is a proof of the Boltyanski-Hadwiger

*Illumination Conjecture for wide spindle convex bodies* generalizing the relevant well-known result of Schramm (1988). [37] K. Bezdek, *The problem of illumination of the boundary of a convex body by affine subspaces*, *Mathematika*, Volume 38 (1991), Pages 362-375.

[107] K. Bezdek, *Illuminating spindle convex bodies and minimizing the volume of spherical sets of constant width*, *Discrete and Computational Geometry*, Volume 47, Issue 2 (2012), Pages 275-287.

(5) *Shortest Closed Billiard Trajectories - Towards a Solution of a Problem of Zelditch (2000)*:

The main result of [27] is that each plane convex body of constant width  $\frac{1}{2}$  is a translation cover for the class of plane closed curves of length 1, and in fact these are the convex translation covers of minimum perimeter for this class. The proof is reduced to minimize the lengths of billiard triangles in plane convex bodies of constant width  $\frac{1}{2}$ . [27] has become a prototype of similar results on translation covers and has led to the following generalization published in [103]. According to the main theorem of [103], any convex body in  $d$ -dimensional Euclidean space ( $d > 1$ ) possesses at least one shortest closed (generalized) billiard trajectory, and, moreover, any of its shortest closed (generalized) billiard trajectories is of period at most  $d + 1$ . Furthermore, the following is proved in [103]: any of the shortest closed (generalized) billiard trajectories of an arbitrary wide spindle convex domain is 2-periodic. In fact, [103] proves the analogue result for  $\epsilon$ -rounded wide spindle convex domains as well. *These theorems give a partial answer to the longstanding question of Zelditch on characterizing convex domains whose shortest closed billiard trajectories are of period 2.*

[27] K. Bezdek and R. Connelly, *Covering curves by translates of a convex set*, *The American Mathematical Monthly*, Volume 96, Issue 9 (1989), Pages 789-806.

[103] D. Bezdek and K. Bezdek, *Shortest billiard trajectories*, *Geometriae Dedicata*, Volume 141, Issue 1 (2009), Pages 197-206.

## 5 Current CCDG Research Group Members

**Ph.D. students:** I. Ivanov (2018- ), N. Robock (2019- ), F. Firoozi (2023- ).

**Researchers:** K. Bezdek, T. Bisztriczky, T. Dinh, F. Fodor, C. Laflamme, R. Woodrow, Y. Zinchenko.

## 6 Supervision of Students and Postdoctoral Fellows

*Supervision of undergraduate research students:*

Under a joint support of NSF (USA) and the Mathematics Department at Cornell University gifted undergraduates participate in 8-week long summer research programs. In the summers of 1995, 1996, 1999 and 2001 Bezdek has supervised the research work of groups of 4-5 undergraduates. *Jade Vinson*, a member from the 1996 REU group of Bezdek, received the 1998 Frank and Bennie Morgan Prize of AMS-MAA-SIAM for his REU research results on fractals, sphere packing and convexity. **Further undergraduate research students supervised:** Ori Cohen (September-December, 2010: U of C and July-August, 2011: Fields Inst., Toronto), Dawson Kennedy (September-December, 2010: U of C), Victoria Labute (September-December, 2010: U of C and July-August, 2011: Fields Inst., Toronto), Duygu Calik (September-December, 2010: U of C), Balázs Szalkai (May-August, 2011: Univ. of Pannonia, Veszprém, Hungary), Samuel Reid (May-August, 2012: U of C), Nóra Frankl (May-August, 2013: Eötvös Loránd Univ., Budapest, Hungary), Michael Oliwa (May-August, 2016: U of C), Heather Berringer (May-August, 2017: U of C), Braden Strachan (May-August, 2019: U of C and May-August, 2020: U of C).

*Supervision of M.Sc. students:*

Károly Böröczky Jr. (1987-1988: Eötvös Loránd Univ., Budapest, Hungary ), Tamás Hausel (1994-1995: Eötvös Loránd Univ., Budapest, Hungary), Bouchra M. Sabbagh (2005-2007: U of C), Wes Maciejewski (2006-2007: U of C), Victoria Labute (2011-2014: U of C), Melanie Foerster (2014-2016: U of C), Michael Oliwa (2016-2018: U of C), Nathan Robock (2017-2019: U of C), Braden Strachan (2021-2023).

*Supervision of Ph.D. students:*

Tibor Ódor (1988-1991: Eötvös Loránd Univ., Budapest, Hungary), László Szabó (1991-1995: Eötvös Loránd Univ., Budapest, Hungary), István Talata (1995-1997: Eötvös Loránd Univ., Budapest, Hungary), Balázs Visy (2000-2002: Eötvös Loránd Univ., Budapest, Hungary), Endre D. Kiss (2001-2004: Eötvös Loránd Univ., Budapest, Hungary), Márton Naszódi (2003-2007: U of C), Zsolt Lángi (2004-2008: U of C), Peter Papez (2003-2009: U of C), Máté Salát (2008-2009: U of C), Ryan Trelford (2009-2014: U of C), Muhammad A. Khan (2013-2017: U of C), Illya Ivanov (2018- : U of C), Nathan Robock (2019- : U of C), Federico Firoozi (2023- : U of C). *Muhammad A. Khan* has won the

Vanier Scholarship (2014-2017) (which was the first to be awarded to the Department of Mathematics and Statistics at U of C), the Killam Scholarship (2014-2016), as well as the Alberta Innovates Doctoral Award (2015-2017).

*Supervision of Postdoctoral Fellows:*

Tsasa Lusala (September, 2010-April, 2011: U of C), Viktor Vigh (July-December, 2011: Fields Inst., Toronto), István Szalkai (May-December, 2014: Univ. of Pannonia, Veszprém, Hungary), Ryan Trelford (January-March, 2015: U of C), Márton Naszódi (May-December, 2013 and May-December, 2016: Eötvös Loránd Univ., Budapest, Hungary), Zsolt Lángi (May-December, 2015, and May-December, 2017: Univ. of Tech., Budapest, Hungary).

## 7 Research Grant Support

1989 - 1991: Principal Investigator, Hungarian National Science Foundation (OTKA), “Structural geometry of discrete systems”;

1992 - 1994: Principal Investigator, Hungarian National Science Foundation (OTKA), “Illumination problems”;

1995 - 1998: Principal Investigator, Hungarian National Science Foundation (OTKA), “Geometric convexity in the theory of packing and covering”;

1997 - 1998: Principal Investigator, Hungarian Ministry of Education, “Discrete, convex, finite and differential geometry”;

1999 - 2000: Principal Investigator, Hungarian Ministry of Education, “Combinatorial geometry”;

1999 - 2002: Principal Investigator, Hungarian National Science Foundation (OTKA), “Discrete geometry and geometric convexity I”;

2003 - 2006: Principal Investigator, Hungarian National Science Foundation (OTKA), “Discrete geometry and geometric convexity II”;

2004 - 2009: Principal Investigator, Natural Sciences and Engineering Research Council of Canada Discovery Grant, “Sphere Arrangements”;

2006 - 2008: Co-Leader, Collaborative Research Group grant of the Pacific Institute for the Mathematical Sciences, “Geometric and Harmonic Analysis”;

2009 - 2014: Principal Investigator, Natural Sciences and Engineering Research Council of Canada Discovery Grant, “Selected Topics in Discrete Geometry I”;

2014 - 2019: Principal Investigator, Natural Sciences and Engineering Research Council of Canada Discovery Grant, “Selected Topics in Discrete Geometry II”;

2019-2024: Principal Investigator, Natural Sciences and Engineering Research Council of Canada Discovery Grant, “Arrangements of Convex Bodies - the Discrete Geometric Side”.

## 8 Visiting Professor Positions

Aug., 1985 - Aug., 1986: Visiting Assistant Professor, Cornell University, Department of Mathematics, Ithaca, New York, USA;

Aug., 1988 - May, 1989: Visiting Associate Professor, Cornell University, Department of Mathematics, Ithaca, New York, USA;

May, 1989 - Aug., 1989 and June, 1991 - Aug., 1991: Visiting Associate Professor, The University of Calgary, Department of Mathematics and Statistics, Calgary, Alberta, Canada;

Dec., 1991 - Jan., 1992: Visiting Professor, Universität Dortmund, Fachbereich Mathematik, Dortmund, Germany;

Apr., 1992 - June, 1992: Visiting Professor, Justus-Liebig-Universität, Mathematisches Institut, Giessen, Germany;

Apr., 1993 - June, 1993: Visiting Professor, Technical University of Eindhoven, Department of Computer Science and Discrete Mathematics, Eindhoven, The Netherlands;

Aug., 1993 - Dec., 1996, July, 1997 - Aug., 1997, July, 1998 - Dec., 1998, July, 1999 - Aug., 1999,

July, 2000 - Aug., 2000, and July, 2001 - Aug., 2001: Visiting Professor, Cornell University, Department of Mathematics, Ithaca, New York, USA;

Sept., 2001 - Dec., 2001: Visiting Professor, University of Texas at Austin, Department of Mathematics, Austin, Texas, USA;

Dec., 2001 - Jan., 2002: Visiting Professor, Universität Dortmund, Fachbereich Mathematik, Dortmund, Germany;

Sept., 2002 - May, 2003: Visiting Professor, Cornell University, Department of Mathematics, Ithaca, New York, USA.