

FACULTY OF ARTS  
DEPARTMENT OF PHILOSOPHY  
**PHIL 479 — “Logic III”**  
**PHIL 679.01 — “Gödel’s Incompleteness Theorem”**  
**Winter Term 2012**

**Course Outline**

Instructor: **Richard Zach**  
Office: 1254 Social Sciences  
Office Hours: MW 4-5 (tentative) or by appointment  
Phone: (403) 220-3170      email: rzach@ucalgary.ca  
Lectures: **MW 5:00-6:15**  
**1253 Social Sciences**

**Course Description**

This course is a complement to and in part a continuation of Phil 379 (Logic II). Specifically, we will focus on two famous theorems of symbolic logic due to Kurt Gödel: The Incompleteness Theorems. The first of these states, roughly, that every formal mathematical theory, provided it is sufficiently expressive and free from contradictions, is incomplete in the sense that there are always statements (in fact, true statements) in the language of the theory which the theory can’t prove.

In order to prove the Incompleteness Theorem, we’ll need to study the expressive power of formal languages and axiomatic theories—this is an important and exciting area in itself. This investigation will lead us naturally to a topic familiar from Logic II, i.e., computability. In Logic III, however, we’ll approach computability not via Turing machines, but via the notion of a recursive function. (We will prove, however, that both notions coincide.)

**Prerequisites and Preparation**

Logic II (PHIL 379) is a prerequisite for this course, although I’ll make an effort not to rely on this.

It can’t hurt to review the material from PHIL 379, especially Chapters 1, 2, 9, 10, 12 of Boolos, Burgess, and Jeffrey.

**Required Text**

Peter Smith, *An Introduction to Gödel’s Theorems*, Cambridge University Press, 2007

## Requirements and Evaluation

The course requirements are: A “diagnostic” homework assignment (5%), four homework assignments (60%—15% each), and a final project (30%—25% paper, 5% presentation). There will be no exams. You must submit all four assignments and the final project. Graduate students registered in Phil 679.01 will solve additional problems on the homework assignments and will take on more ambitious final projects.

The final project will consist in a paper (approx. 2500 words; 3000 words for graduate students) on a topic of your choice related to the material covered in the course. You will give an oral presentation of your paper in the final week of class. Possible projects include: working out a proof only sketched in class in detail, presenting material from a chapter of the book not covered in class, researching a logical topic related to the course, writing a philosophical paper on the importance of Gödel’s theorems. The amount and quality of independent research on your part (or lack thereof) will be taken into account in assigning a grade.

Class participation counts for the remaining 5% of your grade. If you are shy and do not want to speak in class, 4 substantive, serious posts over the course of the term on the online discussion board will earn you an A for participation. Only posts made before the last day of class count. If all your posts are made within a 7 day period, you will receive a maximum credit of 2 grade points for them.

On each problem on an assignment and on the paper and presentation parts of the final project you will receive a letter grade reflecting the level of mastery of the material shown by the work you submit. According to the *Calendar*, letter grades are defined as follows:

- A** Excellent—superior performance, showing comprehensive understanding of subject matter. (Your solution to an assigned problem shows that you understand the problem and how to solve it; the solution is complete and rigorously correct, and is reasonably direct and elegant.)
- B** Good—clearly above average performance with knowledge of subject matter generally complete. (You understand the problem and give a complete solution, although there may be minor gaps in the proof, or the solution is correct but circuitous.)
- C** Satisfactory—basic understanding of the subject matter. (You understand what the question is asking but your solution contains significant errors or gaps.)
- D** Minimal pass—marginal performance. (You show some knowledge of what is asked, but you don’t come near a solution.)
- F** Fail—Unsatisfactory performance. (It is not clear that you understand what the question is asking, or your proposed solution goes completely in the wrong direction.)

The correspondence of letter grades with grade points is defined in the *Calendar* (A = 4, B = 3, C = 2, D = 1, F = 0). “Slash” grades are possible with grade point values 0.5 below the higher grade (e.g., A/B = 3.5).

In computing your final grade, your marks will be converted to grade points and averaged according to the weights given above. The final grade will be the letter grade corresponding to the weighted average of your assignments, exams, and participation plus a margin of 0.2. For the final grade, +’s and –’s are possible, too; as defined in the *Calendar*, +/- adds/subtracts 0.3 grade points. In other words, a course average of 3.8 or higher receives an A; at least 3.5 and less than 3.8, an A–; at least 3.1 and less than 3.5, a B+; at least 2.8 and less than 3.1, a B; and so on. (This means that for two A’s and an A/B you still get an A overall; for two A’s and a B, an A–.) There is no D– grade; to earn a D you require a course average of at least 0.8. The A+ grade is reserved for “truly outstanding” performance.

## Assignments and Policies

*Late work and extensions.* Assignments handed in late will be penalized by the equivalent of one grade point per calendar day. If you turn an assignment in late, you must give it to me personally or put it in the department drop-box (it will then be date-stamped by department staff). Note that the drop-boxes are cleared at 4 pm, the department closes at 4:30 pm on weekdays and *is closed Saturdays and Sundays*.

*Collaboration.* Collaboration on homework assignments is encouraged. However, you must write up your own solutions, and obviously you must not simply copy someone else’s solutions. You are also required to list the names of the students with whom you’ve collaborated on the assignment. **If you collaborate without following these instructions, it constitutes cheating.**

*Plagiarism.* You might think that it’s only plagiarism if you copy a term paper off the Internet. However, you can also plagiarize in a logic course, e.g., by copying a proof verbatim from the textbook (and only making the necessary changes to apply it to the assigned problem.) The point of logic problems which are similar to the proofs in the text is to make you work through those proofs, understand them, and then prove a similar result on the homework. Hence, all homework solutions must be in your own words; copying or paraphrasing closely from the text will be treated as plagiarism and results in a failing grade on the problem; in egregious cases also in a report to the Dean.

*Checking your grades and reappraisals of work.* University policies for reappraisal of term work and final grades apply (see the *Calendar* section “Reappraisal of Grades and Academic Appeals”). In particular, term work (homework assignments, final paper) will only be reappraised within 15 days of the date you are advised of your marks. Please keep track of your assignments (make sure to pick them up in lecture or in office hours) and your marks (check them on the website) and compare them with the graded work returned to you.

## Course Website

A course website on U of C’s BlackBoard server has been set up. You should be automatically registered on the first day of class if you’re registered in the class. To access the BlackBoard site,

you can either go directly to [blackboard.ucalgary.ca](http://blackboard.ucalgary.ca) and log in with your UCIT account name and password, or you can access it through the myUofC portal at [my.ucalgary.ca](http://my.ucalgary.ca).

***Please log on at least once by the end of the second week of classes.***

If you are not registered in the course on the first day of class, you will be added to the website within a day of registering.

I will use the email function on BlackBoard to send out important notices. Therefore, ***please make sure your email address in BlackBoard is current.*** For instructions on how to update your email address, see [elearn.ucalgary.ca/blackboard/email](http://elearn.ucalgary.ca/blackboard/email).

## **Tentative Syllabus and Due Dates**

This is a tentative syllabus to give you a rough idea what parts of the book we will cover when. Due dates are subject to change.

**Week 1: Introduction, Review** (Jan 9, 11). Chapters 1–3

*Learning goals:* Understanding enumerability, effective enumerability, decidability; axiomatic theories and their properties.

**Week 2: Expressing and Capturing Numerical Properties** (Jan 16, 18): Chapters 4–6.

*Learning goals:* Understanding how properties and relations of numbers can be expressed in the language of arithmetic, and how they can be captured by formal theories. Understanding the connections between expressive power of theories and decidability.

Diagnostic Assignment due.

**Week 3: Formal Arithmetics** (Jan 23, 25): Chapters 8–10

*Learning goals:* Acquaintance with formal theories of arithmetic, facility with proving things in these theories. Understanding induction.

Assignment 1 due

**Week 4: Primitive Recursive Functions** (Jan 30, Feb 1). Chapters 11–12.

*Learning goals:* Understanding primitive recursion, facility with defining primitive recursive functions.

**Week 5: Capturing Primitive Recursive Functions** (Feb 6, 8). Chapter 13

*Learning goals:* Understanding why all p.r. functions are  $\Sigma_1$ , and why all  $\Sigma_1$  functions can be captured in  $\mathbb{Q}$ .

**Week 6: Arithmetization of Syntax** (Feb 13, 15): Chapter 15.

*Learning goals:* Understanding Gödel numbering and why syntactic properties are primitive recursive.

Assignment 2 due

Reading week Feb 18–26.

**Week 7: Incompleteness** (Feb 27, 29). Chapter 16, 17

*Learning goals:* Understanding and proving Gödel's First Incompleteness Theorem.

**Week 8: Diagonalization** (Mar 5, 7): Chapters 19–21

*Learning goals:* Generalizing the proof of the first incompleteness theorem; understanding and applying the diagonalization lemma to get Tarski's Theorem about the undefinability of truth.

**Week 9: Provability Predicates and the Second Incompleteness Theorem** (Mar 12, 14): Chapters 24–27.

*Learning goals:* Understanding formalized consistency statements and provability conditions. The second incompleteness theorem. Reflection principles.

Assignment 3 due

**Week 10: Recursive Functions** (Mar 19, 21): Chapters 29, 30.

*Learning goals:* Understanding  $\mu$ -recursive functions.

**Week 11: Recursive Functions and Turing Machines** (Mar 26, 28): Chapter 31–34.

*Learning goals:* Understanding why  $\mu$ -recursive functions are Turing computable and vice versa. The Church-Turing Thesis.

Project draft due

**Week 12: Advanced Topics** (April 2, 4).

Assignment 4 due

**Week 13: Student Presentations** (April 9, 11).

Final project due

## INTELLECTUAL HONESTY

Intellectual honesty is the cornerstone of the development and acquisition of knowledge and requires that the contribution of others be acknowledged. As a result, cheating or plagiarism on any assignment or examination is regarded as **an extremely serious academic offence**, the penalty for which may be an F on the assignment and possibly also an F in the course, academic probation, or requirement to withdraw. The University Calendar states that plagiarism exists when:

- the work submitted or presented was done, in whole or in part, by an individual other than the one submitting or presenting the work (this includes having another impersonate the student or otherwise substituting the work of another for one's own in an examination or test);
- parts of the work are taken from another source without reference to the original author;
- the whole work (e.g., an essay) is copied from another source; and/or
- a student submits or presents work in one course which has also been submitted in another course (although it may be completely original with that student) without the knowledge of or prior agreement of the instructor involved.

While it is recognized that scholarly work often involves reference to the ideas, data and conclusions of other scholars, intellectual honesty requires that such references be explicitly and clearly noted. Plagiarism is an extremely serious offence. Plagiarism occurs not only when direct quotations are taken from a source without specific acknowledgement, but also when original ideas or data from the source are not acknowledged. A bibliography is insufficient to establish which portions of the student's work are taken from external sources; footnotes or other recognized forms of citation must be used for this purpose.

## ACADEMIC ACCOMMODATION

It is the student's responsibility to request academic accommodation. If you are a student with a documented disability who may require academic accommodation and have not registered with the Disability Resource Centre, please contact their office at 403-220-8237. Students who have not registered with the Disability Resource Centre are not eligible for formal academic accommodation. You are also required to discuss your needs with your instructor no later than fourteen (14) days after the start of this course.

## FACULTY OF ARTS PROGRAM ADVISING & STUDENT INFORMATION RESOURCES

- Have a question, but not sure where to start? The new Faculty of Arts Program Information Centre is your information resource for everything in Arts. Drop in at SS 110, phone 403-220-3580 or email at [artsads@ucalgary.ca](mailto:artsads@ucalgary.ca). You can also visit the Faculty of Arts website at [arts.ucalgary.ca/undergraduate](http://arts.ucalgary.ca/undergraduate) which has detailed information on common academic concerns.
- For program planning and advice, contact the Student Success Centre at 403-220-5881 or visit them in their new space on the 3rd Floor of the Taylor Family Digital Library.
- For registration (add/drop/swap), paying fees and assistance with your Student Centre, contact Enrolment Services at 403-210-ROCK [7625] or visit them at the MacKimmie Library Block.
- Email Contact Addresses for Students Union Representatives for the Faculty of Arts: [arts1@su.ucalgary.ca](mailto:arts1@su.ucalgary.ca), [arts2@su.ucalgary.ca](mailto:arts2@su.ucalgary.ca), [arts3@su.ucalgary.ca](mailto:arts3@su.ucalgary.ca), [arts4@su.ucalgary.ca](mailto:arts4@su.ucalgary.ca)

## FREEDOM OF INFORMATION AND PROTECTION OF PRIVACY (FOIP) ACT

The University of Calgary is under the jurisdiction of the provincial FOIP Act in all aspects of its operations as a publicly funded institution. The Department of Philosophy ensures the student's right to privacy by requiring all graded assignments, papers, and exams be returned to the student directly from the instructor unless other arrangements have been made in writing and approved by the Department Administrator.

### **SAFEWALK: PROMOTING CAMPUS SAFETY AND AWARENESS**

Twenty four hours a day, seven days a week, Safewalk volunteers walk people safely to their destination on campus. This service is free and available to students, staff, and campus visitors. Safewalks are done in male/female pairs. The volunteers walk anywhere on campus (including McMahon Stadium, Health Sciences, Student Family Housing, the Alberta Children's Hospital and the University LRT station). To request a Safewalk volunteer to walk with you, call 403-220-5333 (24 hours a day/seven days a week/365 days a year) or use one of the Help Phones located across campus (they are not just for emergencies).